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No. XII.

POPULAR ESSAYS

ON THE

MOVEMENTS OF THE ATMOSPHERE.

COMPILED UNDER THE DIRECTION OF
BRIG. & BVT. MAJ. GEN'L W. B. HAZEN,
CHIEF SIGNAL OFFICER OF THE ARMY.

BY

PROFESSOR WILLIAM FERREL.

PUBLISHED BY AUTHORITY OF THE SECRETARY OF WAR.

WASHINGTON:
OFFICE OF THE CHIEF SIGNAL OFFICER.
1882.

UNITED STATES OF AMERICA; - Signal Service.
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TABLE OF CONTENTS.

I. INTRODUCTION.

II. Paper No. 1.—AN ESSAY ON THE WINDS AND CURRENTS OF THE OCEAN. (From the "Nashville Journal of Medicine and Surgery," 1856.)

III. Paper No. 2.—THE MOTIONS OF FLUIDS AND SOLIDS RELATIVE TO THE EARTH'S SURFACE. (American Journal of Science for 1861.)

IV. Paper No. 3.—CAUSE OF LOW BAROMETER IN THE POLAR REGIONS AND IN THE CENTRAL PART OF CYCLONES. (Nature, July, 1871.)

V. Paper No. 4.—RELATION BETWEEN THE BAROMETRIC GRADIENT AND VELOCITY OF THE WIND. (American Journal of Science, November, 1874.)

VI. Paper No. 5.—METEOROLOGICAL RESEARCHES, PART II., CYCLONES, TORNADOES, AND WATER-SPOUTS. (American Journal of Science, July, 1881.)

INTRODUCTION.

The papers that make up this volume were published originally by Professor Ferrel, one in the "Nashville Journal of Medicine and Surgery," and three in the "American Journal of Science." They are here gathered together for the first time, and in order that they may be in convenient form for all who may wish to study or consult them.

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NO. 1.

AN ESSAY ON THE WINDS AND THE CURRENTS OF THE OCEAN,

BY W. FERREL, A. M., OF NASHVILLE, TENNESSEE.

[Reprinted from "Nashville Journal of Medicine and Surgery," Vol. xi., Nos. 4 and 5, October and November, 1856.]

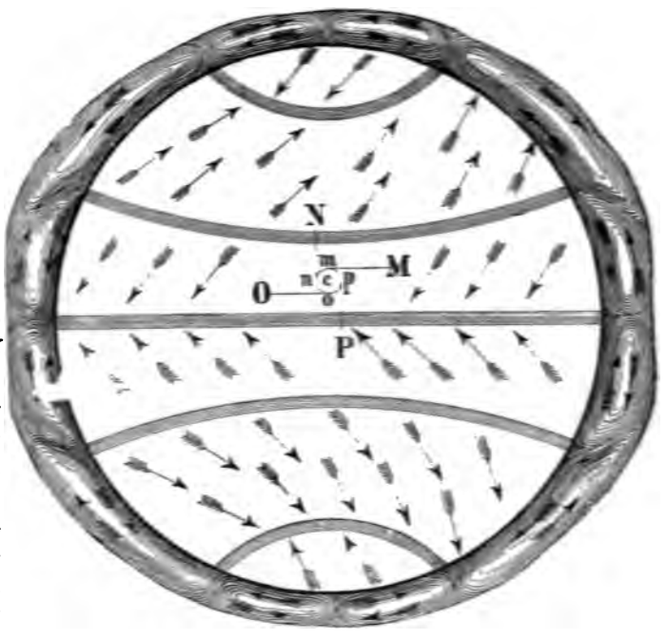
Introduction.—The earth is surrounded on all sides by an exceedingly rare and elastic body, called the atmosphere, extending with a diminishing density to an unknown distance into space, but pressing upon the earth with a force equal to that of a homogeneous atmosphere five and a half miles high. It is also partially surrounded by the ocean, which is of a very variable depth, and known to be, in many places, more than four miles. If the specific gravity of the atmosphere and of the ocean were everywhere the same, all the forces of gravity and of pressure which act upon any part of them, would be in exact equilibrium, and they would forever remain at rest. But as some parts of the earth are much warmer than others, and air and water expand and become rare as their temperature is increased, their specific gravities are not the same in all parts of the earth, and hence the equilibrium is destroyed, and a system of winds and currents is produced. It is proposed in this essay to inquire into the effects which are produced, both in the atmosphere and in the ocean, by this disturbance of equilibrium, and by means of a new force which has never been taken into account in any theory of winds and currents, to endeavor to account for certain phenomena in their motions, which have always been a puzzle to meteorology and hydrology. As there are some uncertain data connected with the subject, such as the amount of the disturbing force, the effects of continents, friction, etc., which render a complete solution of the problem impracticable, we shall aim at giving a popular explanation of observed phenomena rather than a complete solution of the problem; yet we shall give the result of some calculations, based upon known data, or at least upon very reasonable hypothesis, which will show that the causes which we have given are adequate to the effects which are attributed to them. We shall divide the subject into two parts, and treat, first, of the winds, and secondly, of the currents of the ocean.

The motions of the atmosphere.—From about the parallels of 28° on each side of the equator the winds on the ocean, where they are not influenced by any local causes, blow steadily towards the equator, having also a western motion, producing what are called the north-east and south-east trades. At the meeting of these currents near the equator there is a calm called the equatorial calm-belt, or doldrums, where the air rises up and flows in the upper regions towards the poles, until it arrives near the parallels of 28°, where it is met by an upper current flowing from the poles. The meeting of these upper currents produces an accumulation of atmosphere from under which the air flows out in both directions on account of the increased pressure; a strong and steady current, as we have seen, towards the equator, and another not so strong and somewhat variable over the middle latitudes, towards the poles, having at the same time an eastern motion, and producing what are called the passage-winds. As this current flows at the surface towards the poles, it gradually rises up and returns in the upper regions towards the equator, meeting the upper current from the equator near the tropics as has been stated.

Such are the general motions of the atmosphere as laid down by Lieutenant Maury, and as represented in his diagram of the winds.* But there are numerous observations which have been made in very high latitudes both in the north and the south, which show that the currents flowing over the middle latitude *towards* the poles, do not extend to the poles, but that the atmosphere above a certain latitude, has a tendency to flow *from* the poles, producing another meeting of the air at the surface near the polar circles similar to the one at the equator, except that the currents are comparatively feeble and consequently the belt of meeting not so well defined, and that here also the air rises

* Physical Geography of the Sea, page 70.

up and flows each way in the upper regions towards the equator and the poles. "Sir J. Ross has shown that there is a large prevalence of north winds over the southern ones in the middle of North America as low down as latitude 70° and longitude 91° 58′ west, not merely in winter, but in every month in the year. The northern winds were not only more than double as frequent as the southern, but more than double as strong: and he also found southern winds largely to predominate in latitude 77° south."* If then there were no continents, or other local causes of disturbance, the motions of the atmosphere would be as represented in the following diagram, in which the direction of the wind is represented by the arrows, and the external part of which represents the motion of the air in the plane of the meridian. This system, however, is found to prevail on the ocean only, and is very much interfered with in other parts on account of local causes of disturbance, especially in the northern hemisphere, where the uniformity of the earth's surface is most interrupted by land.



The pressure of the atmosphere.—The atmosphere everywhere presses upon the surface of the earth with a force equal to the pressure of a column of mercury of nearly thirty inches in height. This pressure, however, is not the same in all parts of the earth, but varies in different latitudes as well as from local causes. The barometer stands highest about the border of the trade-winds, and is about 0.25 of an inch lower at the equator, and seems to stand the lowest about the polar circle, where a very remarkable depression has been observed in many places. "It is a singular fact," says Mrs. Somerville, (page 268) "discovered by our navigators, that the mean height of the barometer is an inch lower throughout the Antarctic ocean and at Cape Horn, than it is at the Cape of Good Hope, or at Valparaiso." A similar depression has likewise been observed near the sea of Okhotsk in eastern Siberia. It also appears from the tables of the South Sea Exploring Expedition, by Captain Wilkes, that the barometer stands lowest near the polar circle, where it stands at a mean of about twenty-nine inches, and higher both north and south of it. After examining various authorities and all the circumstances connected with this subject, Professor Espy comes to the conclusion, that "there are three belts where the barometer stands below the mean, with almost constant rain and snow—one near the equator, one near the arctic circle, and one near the antarctic circle, and that these belts change their latitudes some degrees with the seasons, following the sun; and also that there are, *certainly* two belts in the outer borders of the trade-winds, where the barometer stands above the mean, and *almost certainly* two regions more—one around the north pole and the other around the south pole—where the barometer stands above the mean." It also appears from a great many observations made at different places on the Atlantic, at the level of the ocean, that the barometer stands more than half an inch lower at the arctic circle, than it does at the outer limit of the trade-winds, and that there is also a considerable depression at the equator.†

The forces concerned in producing the motions of the atmosphere.—There are four principal forces which must be taken into account in a correct theory of the winds. The first arises from a greater specific gravity of the atmosphere in some places than others, on account of a difference of temperature and of the dew-point; for, when it becomes heated or charged with vapor in any place to a greater degree than at others, it becomes specifically lighter, and hence, the equilibrium is destroyed. There is a flowing together, then, of the heavier air on all sides, which displaces the lighter air, and causes it to rise up and flow out in a contrary direction. This is the *primum mobile* of the winds, and all the other forces concerned are dependent upon it for their efficiency. A second force arises from the

* Professor Espy's Third Report on Meteorology, § 101. † See Kaemtz' Meteorology, by C. Walker, page 277.

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phere receives a motion either towards or from the poles, the action of the third force above, causes a deflection of it towards the east, as it moves towards the poles, and towards the west as it moves towards the equator; and as the prime moving cause of the principal currents of the atmosphere has a tendency to cause it to flow towards and from the poles, the general result is, that towards the poles the atmosphere has a motion towards the east, but near the equator towards the west. But from the principle of the preservation of areas, the sum of the products of all the particles of the atmosphere, multiplied into their velocities and their distances from the axis of revolution, cannot be changed by the action of a central force, or by the mutual action of the particles upon each other; hence the sum of the products of each particle into its distance from the axis, and into its relative eastern velocity, must be equal to the sum of similar products, taken with regard to the particles having a relative western motion. But, as the portion of the atmosphere having a relative eastern motion, is nearer to the axis than that which has a relative western motion, and as the part of the atmosphere between the parallels of 30°, comprises one half of the whole, the part having a western motion, inasmuch as it is further from the axis, must be somewhat less than the part comprised between these parallels, in order to make the products equal, unless the relative velocity of the part having an easterly motion is very much greater. Hence, the dividing lines between the portions of the atmosphere having a relative east and west motion, must be within these parallels; and, as the outer limits of the trade-winds depend upon those lines, they must also fall within these parallels; and they are accordingly found to be about the parallels of 28°.

The cause of high barometer about the parallels of 28°, *and the low barometer at the polar circles.*— The greater pressure of the atmosphere at the parallels of 28° than at the equator and the polar circles can only be caused by an accumulation of atmosphere there. This accumulation results, necessarily, from the action of the new force which we have introduced into the theory of the winds. For, as we have seen, all the atmosphere between the parallels of 28° and the poles has, and, according to theory, must have, a general eastern motion; and this gives such a value to the analytical expression of the fourth force, enumerated above, as to cause the atmosphere to recede from the poles toward the equator. But the western motion of the atmosphere between the parallels of 28° gives that expression a negative value there; and, hence, this force causes the atmosphere to recede from the equator, also. This force, then, has a tendency to cause the atmosphere in the upper regions to recede both from the poles and the equator, and to accumulate about the parallels of 28°, and, as it may seem by merely inspecting the expression of this force given above, that for the same value of v, or motion of the atmosphere east or west, this force is much greater toward the poles than it is near the equator, it causes a considerable depression of the atmosphere at the poles, and only a slight one at the equator, as represented in the diagram. The amount of this elevation and depression is not indicated entirely by the barometer, for the height of the barometer depends upon both the height of the atmosphere and its density. Therefore, as the atmosphere is much denser at the polar circles than at the parallels of 28°, on account of its being much colder, the accumulation of atmosphere at these parallels and its depression towards the poles, must be considerable to cause the barometer to stand higher at these parallels than at the polar circles.

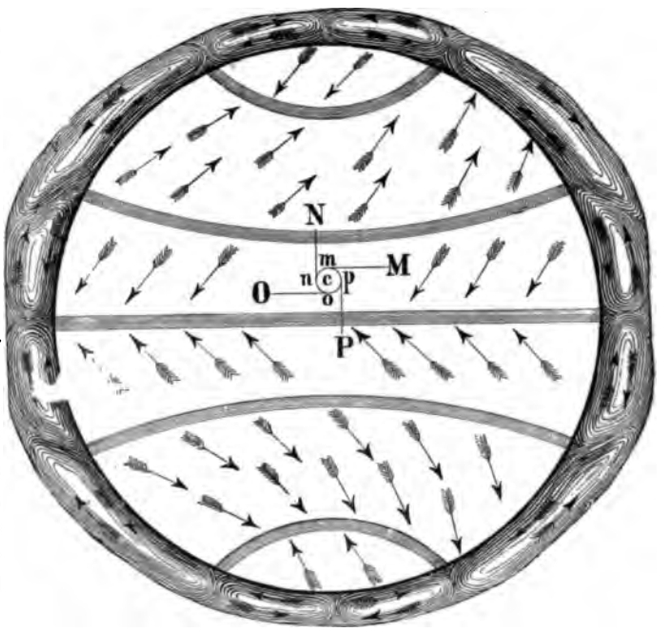
We shall now give the results of some calculations, based upon a reasonable hypothesis, which show that this new force introduced is entirely adequate to produce an accumulation of atmosphere at the parallels of 28°, and a depression of it at the poles to such an amount, that the difference in the height of the barometer at the parallels of 28° and the polar circles, may correspond with observations. As friction is a very important element in calculations of the motions of the atmosphere, and its effect cannot be determined, it would be impossible to calculate the motions of the atmosphere from the forces which act upon it, if they were even accurately known. We will therefore assume certain motions of the atmosphere, which are known not to vary much from observation, upon which we will base our calculations. If we assume that the east and west motions of the atmosphere may be represented by the expression $2 \sin. \frac{3}{2} p \cos. \frac{3}{2} p v$, p being the polar distance of the place, it would make these motions vanish at the poles and at the parallels of 30°, and we have reason to think, would pretty well represent the motions of the atmosphere east and west at all latitudes except near the equator, where it would make it a little too great. Upon this assumption it may be shown by calculations, the results of which only can be given here, that if v, or the maximum east or west motions of the atmosphere, were only ten miles per hour, it would cause a heaping up of the atmosphere near the parallels of 28° which would make the barometer, if the atmosphere were everywhere of the same

density, stand two inches higher here than at the poles. The same hypothesis would also make the depression at the equator only one-ninth that at the poles. If we now suppose that the greater specific gravity alone of the atmosphere at the poles, is sufficient to cause the barometer to stand one inch higher there than at the equator, which would only require a difference of temperature of about 16°, it would still leave a difference in the height of the barometer between the poles and the equator about equal to the observed difference in the southern hemisphere. It would also add a little to the depression of the barometer at the equator, which would make it a little more than one-ninth of that at the poles, and consequently make it correspond with observation. We think therefore it is evident that the observed difference in the height of the barometer in different latitudes, is owing to the joint effect of a gradual increase of specific gravity from the equator to the poles, and of a heaping up of the atmosphere near the parallels of 28°, caused by the action or this new force which we have taken into account.

Explanation of the passage-winds and calm-belts at the limits of the trade-winds.—As the pressure of the atmosphere, on account of its accumulation there, is greatest about the parallels of 28°, this pressure has a tendency to cause it to rush out from beneath both, towards the poles and the equator. If the motions of the atmosphere were as great at the surface of the earth as in the upper regions, the force which causes a heaping up of the atmosphere about the parallels of 28°, would be as great below as in the upper regions, and would prevent the flowing out of the air below towards the poles. But, on account of friction, the eastern motion of the atmosphere cannot be so great at the surface of the earth as above, and consequently the accumulation of atmosphere mentioned above, is caused principally by the upper currents, and the pressure which causes it to flow out below towards the poles, where the barometer, as we have seen, stands much lower, is greater than the force below which causes the accumulation of atmosphere. The lateral pressure then of the atmosphere, and its horizontal motions north and south, depend upon three forces; first, the greater density of atmosphere towards the poles, which has a tendency to cause it to flow at the surface of the earth, from the poles towards the equator; and, secondly, the heaping up of the atmosphere at the outer limits of the trade-winds, which causes it to rush out below, both towards the equator and the poles; thirdly, the action of the force depending upon the east or west motion of the atmosphere, which we have seen, must be greater above than at the surface of the earth. Between the parallels of 28° and the equator, the first two forces combine against the latter, which is small near the equator, and produce a strong and steady current at the suface of the earth towards the equator, which, being combined with the rotatory motion of the earth, gives rise to the trade-winds. Beyond these parallels, the first and third forces are opposed to the second, but it may be seen from the analytical expression of this second force, obtained from our preceding assumption, and which cannot be given here, that this force is very great in the middle latitudes, and consequently it prevails over the two, causing a current towards the poles, which combined with the rotatory motion of the earth, produces the southwest winds in the northern hemisphere, and northwest winds in the southern hemisphere, called passage-winds. This force has its maximum about the parallels of 48°, and above these it decreases rapidly, so that at the polar circles the other two forces begin to prevail over it, and cause a current from the poles. The forces then acting upon the atmosphere at the surface of the earth, causes it to flow in opposite directions, from the parallels of 28° and the poles, and to flow together near the equator and the polar circles. Hence, there is a rising up of the atmosphere at the latter places, and a flowing thence in the upper regions to the former places, where it descends, and thus a system of current is produced as represented by the arrows in the external part of the diagram, which represents a meridional vertical section of the atmosphere. It was shown that about the parallels of 28°, the atmosphere can have no motion east or west, and it has now been shown that these are the parallels also of greatest pressure, whence the currents flow both towards the equator and the poles, consequently there must necessarily be here calm-belts, such as are well known from observation to exist.

We think now, it is manifest, that the introduction of our new force into the theory of the winds, exactly accounts for all the principal motions of the atmosphere, and clears up the difficulties which have heretofore puzzled meteorologists.

Maury's theory of the crossing of the winds.—In order to account for the motions of the winds and other phenomena, Lieutenant Maury advances the theory that there is a crossing of the winds or currents at the calm-belts of the equator and the parallels of 28°; that the currents flowing at the surface towards the equator, cross there, each one becoming the upper current in the other hemisphere



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* Professor Espy's Third Report on Meteorology, p. 101. † See Kæmtz' Meteorology, by C. Walker, page 277.

tendency which the atmosphere has, when, from any cause, it has risen above the general level, to flow to places of a lower level. These two preceding forces generally produce counter-currents. Again, when, from any cause, a particle of air has been put in motion toward the north or south, the combination of this motion with the rotatory motion of the earth produces a third force, which causes a deflection of the motion to the east when the motion is toward the north, and a deflection toward the west when it is toward the south. This is the same as one of the forces contained in La Place's general equations of the tides, the analytical expression of which is $2nur \sin. l \cos. l$; l being the latitude; n, the motion of the earth at the equator; u, the velocity of the particle north or south; and r, the radius of the earth. The fourth and last force arises from the combination of a relative east or west motion of the atmosphere with the rotatory motion of the earth. In consequence of the atmosphere's revolving on a common axis with that of the earth, each particle is impressed with a centrifugal force, which, being resolved into a vertical and a horizontal force, the latter causes it to assume a spheroidal form conforming to the figure of the earth. But, if the rotatory motion of any part of the atmosphere is greater or less that of the surface of the earth, or, in other words, if any part of the atmosphere has a relative eastern motion with regard to the earth's surface, this force is increased, and if it has a relative western motion, it is diminished, and this difference gives rise to a disturbing force which prevents the atmosphere being in a state of equilibrium, with a figure conforming to that of the earth's surface, but causes an accumulation of the atmosphere at certain latitudes and a depression at others, and the consequent difference in the pressure of the atmosphere at these latitudes very materially influence its motions. This force is also expressed by one of the terms of La Place's equations, the analytical expression of which is $2nur \sin. l$; v being the relative eastern or western velocity of the atmosphere.

Hadley's theory.—This theory, which is the commonly received theory of the trade-winds, and with which the reader is no doubt familiar, is based upon the first three forces only given above, no account being taken of the fourth. But as it may be seen from the analytical expressions of the last two forces given above, that the latter is greater than the former, and the east and west motion of the atmosphere depends upon the former, we have reason to suppose that the latter also may have a very considerable effect, and that it should be taken into account in a correct theory of the winds. Accordingly we see that although Hadley's theory furnishes an explanation of the trade-winds, yet it does not account for many other remarkable phenomena in the motions of the atmosphere, but even requires motions to satisfy it entirely at variance with them. According to this theory, there should be a current on the surface of the earth from the pole to the equator in a kind of loxodromic spiral, and a similar counter-current in the upper regions from the equator towards the poles. The barometer also should stand highest at the poles where the air is coldest and most dense, and gradually fall as it is brought nearer the equator. But both of these, as we have shown, are contrary to observation. The position also of the exterior limits of the trade-winds near the parallels of 28°, the flowing of the atmosphere in the upper regions from both sides towards these parallels, and a low barometer near the polar circles, cannot be explained by this theory, and have not been satisfactorily explained by meteorologists. It is true Professor Espy says the highness of the barometer near the parallels of 28° is owing to the flowing over of the atmosphere which rises up at the equator; but this overflow of atmosphere could have no tendency to accumulate at these parallels, since it evidently would flow on gradually towards the poles to supply the draught caused by the flow towards the equator, as this theory requires. He also assigns as a cause of the low barometer about Cape Horn and the antarctic circle, the abundant rains which prevail there and the consequent disengagement of caloric, which rarefies the atmosphere there. But, if the belt of calms and rains near the equator is caused by the barometer's standing lower there than at the outer limit of the trade-winds, which causes a flow of atmosphere there at the surface, and, if rains generally in any region, according to Professor Espy's own theory of clouds and rains, is caused by a low barometer there, and a consequent flowing in from all sides and a rising up of the atmosphere, then the lowness of the barometer in these regions must be the cause of the rains, and not the rains the cause of the lowness of the barometer, as will appear for other reasons.

We shall now undertake to show that all these phenomena, and others connected with storms, which have never been accounted for by any theory, may be satisfactorily accounted for by taking into account the fourth force given above.

Why the outer limits of the trade-winds are near the parallels of 28°.—If from any cause the atmos-

phere receives a motion either towards or from the poles, the action of the third force above, causes a deflection of it towards the east, as it moves towards the poles, and towards the west as it moves towards the equator; and as the prime moving cause of the principal currents of the atmosphere has a tendency to cause it to flow towards and from the poles, the general result is, that towards the poles the atmosphere has a motion towards the east, but near the equator towards the west. But from the principle of the preservation of areas, the sum of the products of all the particles of the atmosphere, multiplied into their velocities and their distances from the axis of revolution, cannot be changed by the action of a central force, or by the mutual action of the particles upon each other; hence the sum of the products of each particle into its distance from the axis, and into its relative eastern velocity, must be equal to the sum of similar products, taken with regard to the particles having a relative western motion. But, as the portion of the atmosphere having a relative eastern motion, is nearer to the axis than that which has a relative western motion, and as the part of the atmosphere between the parallels of 30°, comprises one half of the whole, the part having a western motion, inasmuch as it is further from the axis, must be somewhat less than the part comprised between these parallels, in order to make the products equal, unless the relative velocity of the part having an easterly motion is very much greater. Hence, the dividing lines between the portions of the atmosphere having a relative east and west motion, must be within these parallels; and, as the outer limits of the trade-winds depend upon those lines, they must also fall within these parallels; and they are accordingly found to be about the parallels of 28°.

The cause of high barometer about the parallels of 28°, *and the low barometer at the polar circles.*— The greater pressure of the atmosphere at the parallels of 28° than at the equator and the polar circles can only be caused by an accumulation of atmosphere there. This accumulation results, necessarily, from the action of the new force which we have introduced into the theory of the winds. For, as we have seen, all the atmosphere between the parallels of 28° and the poles has, and, according to theory, must have, a general eastern motion; and this gives such a value to the analytical expression of the fourth force, enumerated above, as to cause the atmosphere to recede from the poles toward the equator. But the western motion of the atmosphere between the parallels of 28° gives that expression a negative value there; and, hence, this force causes the atmosphere to recede from the equator, also. This force, then, has a tendency to cause the atmosphere in the upper regions to recede both from the poles and the equator, and to accumulate about the parallels of 28°, and, as it may seem by merely inspecting the expression of this force given above, that for the same value of v, or motion of the atmosphere east or west, this force is much greater toward the poles than it is near the equator, it causes a considerable depression of the atmosphere at the poles, and only a slight one at the equator, as represented in the diagram. The amount of this elevation and depression is not indicated entirely by the barometer, for the height of the barometer depends upon both the height of the atmosphere and its density. Therefore, as the atmosphere is much denser at the polar circles than at the parallels of 28°, on account of its being much colder, the accumulation of atmosphere at these parallels and its depression towards the poles, must be considerable to cause the barometer to stand higher at these parallels than at the polar circles.

We shall now give the results of some calculations, based upon a reasonable hypothesis, which show that this new force introduced is entirely adequate to produce an accumulation of atmosphere at the parallels of 28°, and a depression of it at the poles to such an amount, that the difference in the height of the barometer at the parallels of 28° and the polar circles, may correspond with observations. As friction is a very important element in calculations of the motions of the atmosphere, and its effect cannot be determined, it would be impossible to calculate the motions of the atmosphere from the forces which act upon it, if they were even accurately known. We will therefore assume certain motions of the atmosphere, which are known not to vary much from observation, upon which we will base our calculations. If we assume that the east and west motions of the atmosphere may be represented by the expression $2 \sin. \frac{3}{2} p \cos. \frac{3}{2} p r$, p being the polar distance of the place, it would make these motions vanish at the poles and at the parallels of 30°, and we have reason to think, would pretty well represent the motions of the atmosphere east and west at all latitudes except near the equator, where it would make it a little too great. Upon this assumption it may be shown by calculations, the results of which only can be given here, that if r, or the maximum east or west motions of the atmosphere, were only ten miles per hour, it would cause a heaping up of the atmosphere near the parallels of 28° which would make the barometer, if the atmosphere were everywhere of the same

density, stand two inches higher here than at the poles. The same hypothesis would also make the depression at the equator only one-ninth that at the poles. If we now suppose that the greater specific gravity alone of the atmosphere at the poles, is sufficient to cause the barometer to stand one inch higher there than at the equator, which would only require a difference of temperature of about 16°, it would still leave a difference in the height of the barometer between the poles and the equator about equal to the observed difference in the southern hemisphere. It would also add a little to the depression of the barometer at the equator, which would make it a little more than one-ninth of that at the poles, and consequently make it correspond with observation. We think therefore it is evident that the observed difference in the height of the barometer in different latitudes, is owing to the joint effect of a gradual increase of specific gravity from the equator to the poles, and of a heaping up of the atmosphere near the parallels of 28°, caused by the action or this new force which we have taken into account.

Explanation of the passage-winds and calm-belts at the limits of the trade-winds.—As the pressure of the atmosphere, on account of its accumulation there, is greatest about the parallels of 28°, this pressure has a tendency to cause it to rush out from beneath both, towards the poles and the equator. If the motions of the atmosphere were as great at the surface of the earth as in the upper regions, the force which causes a heaping up of the atmosphere about the parallels of 28°, would be as great below as in the upper regions, and would prevent the flowing out of the air below towards the poles. But, on account of friction, the eastern motion of the atmosphere cannot be so great at the surface of the earth as above, and consequently the accumulation of atmosphere mentioned above, is caused principally by the upper currents, and the pressure which causes it to flow out below towards the poles, where the barometer, as we have seen, stands much lower, is greater than the force below which causes the accumulation of atmosphere. The lateral pressure then of the atmosphere, and its horizontal motions north and south, depend upon three forces; first, the greater density of atmosphere towards the poles, which has a tendency to cause it to flow at the surface of the earth, from the poles towards the equator; and, secondly, the heaping up of the atmosphere at the outer limits of the trade-winds, which causes it to rush out below, both towards the equator and the poles; thirdly, the action of the force depending upon the east or west motion of the atmosphere, which we have seen, must be greater above than at the surface of the earth. Between the parallels of 28° and the equator, the first two forces combine against the latter, which is small near the equator, and produce a strong and steady current at the suface of the earth towards the equator, which, being combined with the rotatory motion of the earth, gives rise to the trade-winds. Beyond these parallels, the first and third forces are opposed to the second, but it may be seen from the analytical expression of this second force, obtained from our preceding assumption, and which cannot be given here, that this force is very great in the middle latitudes, and consequently it prevails over the two, causing a current towards the poles, which combined with the rotatory motion of the earth, produces the southwest winds in the northern hemisphere, and northwest winds in the southern hemisphere, called passage-winds. This force has its maximum about the parallels of 48°, and above these it decreases rapidly, so that at the polar circles the other two forces begin to prevail over it, and cause a current from the poles. The forces then acting upon the atmosphere at the surface of the earth, causes it to flow in opposite directions, from the parallels of 28° and the poles, and to flow together near the equator and the polar circles. Hence, there is a rising up of the atmosphere at the latter places, and a flowing thence in the upper regions to the former places, where it descends, and thus a system of current is produced as represented by the arrows in the external part of the diagram, which represents a meridional vertical section of the atmosphere. It was shown that about the parallels of 28°, the atmosphere can have no motion east or west, and it has now been shown that these are the parallels also of greatest pressure, whence the currents flow both towards the equator and the poles, consequently there must necessarily be here calm-belts, such as are well known from observation to exist.

We think now, it is manifest, that the introduction of our new force into the theory of the winds, exactly accounts for all the principal motions of the atmosphere, and clears up the difficulties which have heretofore puzzled meteorologists.

Maury's theory of the crossing of the winds.—In order to account for the motions of the winds and other phenomena, Lieutenant Maury advances the theory that there is a crossing of the winds or currents at the calm-belts of the equator and the parallels of 28°; that the currents flowing at the surface towards the equator, cross there, each one becoming the upper current in the other hemisphere

after it crosses the calm-belt at the equator, and then flowing towards the poles until it meets the upper current flowing toward the equator about the parallel of 28°, where there is supposed to be another crossing, each current then becoming again the surface current, and flowing in the same direction as before.* He also makes, by his arrangement, the rains in each temperate zone depend upon the vapor received by the winds in their passage to the equator as a trade-wind in the opposite hemisphere. We think there is no necessity for resorting to such an argument to account for the phenomena of the winds, but that they are all satisfactorily accounted for, as above, by tracing out the effects of well-known forces without resorting to the mysterious agents of magnetism and electricity. Besides, there is no known principle by which two currents can interpenetrate and cross each other without mingling together, and, especially, is there none by which a current saturated with vapor can pass through a dry current and each one afterwards retain its distinctive character of a moist or dry current, which this theory requires.

The fact that Ehrenberg has discovered South American infusoria in the blood-rains and "sea-dust" of the Cape Verde Islands and other places, does not prove the crossing of the winds; for, according to the explanation of the winds given above, there are two currents flowing into each calm-belt, and also two flowing out in opposite directions, as it were from a common reservoir, and, consequently, whatever is carried into these belts from either side flows out again in each direction; and so infusoria in one hemisphere can easily pass to the other without a distinct crossing of the currents. And if even the moist currents of the torrid zone could pass through the dry ones to the temperate zones, they could not produce rain there; for Professor Espy has conclusively shown that no descending current, however saturated with moisture, can ever produce rain.†

Explanation of the winds at the Peak of Teneriffe.—We have stated that the greatest atmospheric pressure is about the parallels of 28°, but these can not be accurately the parallels of the greatest accumulation, for this pressure depends both upon the height of the atmosphere and its density, and, as the density increases gradually with the latitude, there must be an increase of pressure beyond the parallel of greatest accumulation, until the decrease of pressure from the one cause equals the increase from the other. But, as the calm-belts in the upper regions must be where the currents meet, and consequently where there is the greatest accumulation of atmosphere, it follows that the calm-belts are not exactly at the same parallels at the surface of the earth as in the upper regions, but that they incline above toward the equator, as represented in the diagram. And this explains the peculiarity in the winds at the Peak of Teneriffe. This peak stands near the outer limits of the trade-winds, and, as this limit moves north and south with the seasons, the northeast and southwest winds are found to prevail at the base alternately. But at the top of the peak the southwest winds always prevail, because, when the calm-belt is furthest north, it still leaves the top of the peak north of it, where the southwest winds prevail, while the northwest trades are blowing below. When this belt occupies its most southern position, it leaves both the top and bottom of the peak north of it, and, consequently, the southwest wind blows both at the top and the bottom. In the fall, as the calm-belt moves south, more of the peak gradually becomes north of the calm-belt, and hence the southwest winds, which always prevail at the top, should gradually descend lower on the peak until they reach the base, which is exactly in accordance with observation.

The effects of continents.—If the surface of the earth were all covered by the sea, uninterrupted by continents, the trade-winds and passage-winds, and also the calm-belts, would extend, without any interruption, entirely around the earth. But continents, and especially high mountain ranges, seem to have a very material effect in changing this regular system of winds. Thus the high table-lands and mountain ranges in Mexico and the western part of the United States, seem to turn the westward current of the trade-winds on the Caribbean sea and the Gulf of Mexico northward over the parallel of the calm-belt into the United States, where it arrives at the latitudes where the atmosphere has a general tendency to flow eastward, and thus a kind of aerial gulf-stream is produced. This is evident not only from observations on the general directions of the winds in the Gulf of Mexico and the United States, but also from the observed routes of storms, which must be governed very much by the general movements of the part of the atmosphere in which they occur. Instead of the regular trade-winds from the northeast in the Caribbean sea, the prevailing course of the lower current, is from a point south of east instead of north of east.‡ It is also found from observations at Barbadoes

* See "Physical Geography of the Sea," § 106. † Third Report on Meteorology, §§ 68, 69. ‡ See W. C. Redfield, Esq., on three several hurricanes, etc., in Silliman's Journal, second series, vol. 1., page 13.

that, while the eastern winds are most prevalent, the southeast winds greatly predominate over the northeast ones. Of a great many hurricanes, also, which had their origin in the Atlantic, east of the Caribbean sea, and whose routes have been determined by Mr. Redfield, nearly all moved in a direction north of west, until they arrived at the longitude of Florida or the Gulf of Mexico, where they curved around towards the north, and after passing the parallel of the calm-belt, towards the northeast, in the direction of Newfoundland and the northern part of the Atlantic. And this is exactly the route we would suppose the westward currents of the lower part of the atmosphere, interrupted by the high mountain ranges of Mexico, would take. But on the west coast of North America, the eastward current of the northern part of the Pacific, impinging against the range of the Rocky mountains, is turned down towards the equator, and hence the prevailing direction of the wind on the Pacific, west of Mexico, is from the northwest. The eastern coast of Africa also seems to have a similar effect upon the westward current of air in the Indian ocean; for the hurricanes which originate in that ocean, on approaching that coast, are turned southward and finally towards the southeast into the southern ocean. The typhoons, also, of the China sea seem to be influenced in a similar manner by the eastern part of Asia. These changes of the general directions of the wind which prevail on the open ocean must be caused by the continents.

Hurricanes and storms.—Hurricanes are generally supposed to be produced by the meeting of adverse currents, which produce gyratory motions of the atmosphere at the place of meeting. That they may receive their origin and first impulse in this way, we think is very probable; but that violent hurricanes, extending over a circular area of nearly one thousand miles in diameter, and continuing for ten days, and proceeding with increasing violence from the torrid zone to high northern latitudes, depends upon any primitive impulse alone, we think is very improbable. For if even any part of the atmosphere should receive such an impulse as to produce a most violent hurricane, friction would soon destroy all motion and bring the atmosphere to rest. Besides, no gradually accelerated motion can depend upon a primitive impulse alone, even where there is no friction. Hurricanes then, and all ordinary storms, must begin and gradually increase in violence by the action of some constantly acting force, and when this force subsides, friction brings the atmosphere to a state of rest. This force may be furnished by the condensation of vapor ascending in the upward current in the middle of the hurricane, in accordance with Professor Espy's theory of storms and rains. According to this theory, all storms are produced by an ascending current of warmer atmosphere saturated with moisture, and this current is kept in motion by the continual rarefaction of the atmosphere above by means of the caloric given out of the vapor which is condensed as it ascends to colder regions above. Therefore, as long as this ascending current can be supplied with air saturated with vapor this continual rarefaction must take place, and also the ascent of the air in the middle of the hurricane and the consequent rush of air from all sides to supply its place. If, then, all the lower stratum of atmosphere over a large district were saturated with vapor, without some disturbing cause, it might remain undisturbed; but if from any cause an ascending current is produced, either by local rarefaction of air by means of heat, or by the meeting of two adverse currents, which produces a gyratory motion and consequent rarefaction in the middle on account of the pressures being taken away by the centrifugal force, as soon as the air below, saturated with vapor, ascends to the colder regions above, the vapor is condensed and the caloric given out continues to rarefy it so long as the ascending column is supplied with moist air, and consequently the surrounding colder air presses in below from all sides, and thus a hurricane of more or less violence is produced and kept up for ten or twelve days, moving with the general direction of the motion of the atmosphere where it occurs. The violence then of the hurricane, and also its duration, depends upon the quantity of vapor supplied by the currents flowing in below. Hence, it is, that the tropical hurricanes which originate in the Atlantic, east of the Caribbean sea, do not abate their violence until they reach a high northern latitude where the atmosphere is cold and dry.

The cause of the gyratory motions of hurricanes.—It has been well established by Redfied, Reid, Piddington, and others, that all hurricanes and ordinary storms have a gyratory motion around a center, and that these gyrations in the northern hemisphere, are from right to left *against* the hands of a watch, but in the southern hemisphere from left to right *with* the hands of a watch. There are some, however, amongst whom is Professor Espy, who deny the gyratory character of storms entirely, and contend that there is only a rushing of the air from all sides below towards a centre without any gyration. We think that this gyratory character of storms has been too well established to admit of

any doubt. No one, however, has ever given any satisfactory reason why these gyrations in the one hemisphere are always sinistrorsal and in the other dextrorsal. It is true, Mr. Redfield* endeavors to account for the sinistrorsal gyrations of the hurricanes and storms which proceed from the Gulf of Mexico towards Newfoundland, by means of the peculiarities of the aerial currents in the region of the Gulf and the adjacent coast of the Pacific. But if there are the same kind of gyrations over the whole hemisphere, it is evident that the cause which produces them must be as extended as the hemisphere itself. It has also been suggested that this tendency to a distinct kind of gyration in each hemisphere, may be owing to the magnetism of the air.†

We shall now undertake to show that there cannot be a rushing of air from all sides towards a center, on any part of the earth except at the equator, without producing a gyration, and that the tendency to a distinct kind of gyration in each hemisphere, is owing, neither to any peculiarities of the winds or aerial currents, nor to the mysterious agent of magnetism, but that it results, as a necessary consequence, from the action, upon the atmosphere, of the four forces which we have taken into consideration in the first part of this essay.

It has been shown that when a particle of air receives a motion toward the poles it is deflected toward the east, as in the passage-winds, but when it receives a motion toward the south, there is a force which also turns it towards the west, as in the trade-winds. It has likewise been shown that when the air has a relative motion east, it has a tendency, on account of the greater centrifugal force, to move also towards the south, but that when it has a relative motion west, it has a tendency, on account of the diminished centrifugal force, to move also towards the north. If, then, we suppose that the air at *M*, *N*, *O*, and *P* on the diagram (page 8), has a tendency, on account of rarefaction or for any other reason, to flow towards *c*, from what has been stated, the air at *M* would not move equally towards *c*, but would be deflected northward a little towards *m*. In like manner the air at *N* would move towards *n*; that at *O* towards *o*; and that at *P* towards *p*. Hence it is evident that when there is any force which tends to make the surrounding air flow towards a center, the resultants of all the forces which act upon it must cause it to receive a gyratory motion, and that this motion in the northern hemisphere must be sinistrorsal, but in the southern hemisphere the contrary.

It may be observed here that these gyrations are not circular but spiral, gradually approaching the center; for the forces which tend to produce these gyrations depend for their efficacy upon a motion from all sides towards the center. First, the forces of which we have already treated tend to give the atmosphere a gyratory motion as soon as it begins to converge towards a center; and secondly, these gyrations, however slight, being once produced, the centripetal force, which causes the air to flow towards the center, accelerates these gyrations as they approach the center, upon the principle of the preservation of areas, just as the gyrations of a body, when it is made to gyrate around another by means of a string, are rapidly accelerated as the string becomes shorter. Hence, if the first of these forces be only sufficient to produce a very slight gyration, the latter or centripetal force may cause very rapid gyrations near the center. And it is only upon this principle that the rapid motion of the air in a hurricane can be produced, and any theory which does not take this principle into account is defective. This centripetal force is caused by the superior pressure of the denser atmosphere on the borders of the hurricane or storm, and consequently prevails only in the lower part of the atmosphere. In the upper regions it has a tendency to recede from the center for two reasons; first, on account of the gyratory motion which it has received below in approaching the center, which it still, in some measure, retains after ascending to the regions above, where the surrounding pressure does not prevail, and consequently the centrifugal force resulting from the gyrations, causes it to recede from the center; secondly, because the ascending current causes an accumulation of atmosphere above the general level, which gives it a tendency also to flow out in all directions from the center. These motions, however, are not distinctly towards and from the center, but in spirals, so that the currents below may be at right angles with the currents above; and hence it is that in our ordinary storms attended with rain, the clouds in the lower part of the atmosphere frequently move in a direction at right angles with the direction of those above.

It has been stated that the gyrations below approach the center in a spiral, but this approach must be slow towards the center; for, at a certain distance from the center, the gyrations become so rapid that the centrifugal force nearly equals the centripetal, produced by the external pressure of the

* Silliman's Journal, second series, vol. II., page 325. † Maury's Physical Geography of the Sea, § 224.

atmosphere, and then the further approach towards the center in a great measure ceases, and consequently the force which produces the gyrations. Hence, at a certain distance from the center the hurricane has the greatest violence, and within this circumference, friction in a great measure destroys the gyrations, so that the middle of our most violent hurricances is a calm. The extent of this calm is a circle, varying generally from five to thirty miles in diameter.

If we examine the analytical expressions of the forces which produce these gyrations, we will see that at the equator they have no value, and hence no hurricane can have its origin exactly on the equator. Accordingly, of all the hurricanes which have originated within the tropics, none have been traced back to the equator, but always to some region from 10° to 20° from it.

The reason why the hurricanes which originate east of the Caribbean sea pass northward to the east of the United States, may be owing to the direction of the wind here and on the Caribbean sea, which generally blows north of west, as has been observed in the former part of this essay. If the general direction of the trade-winds prevailed here, they would be carried on towards the equator, as those without doubt are which originate at other places in the same latitude.

We come now to the second part of our subject, the currents of the ocean.

The general motions of the ocean.—Inasmuch as the atmosphere and the ocean are both fluids somewhat similarly situated, except that the latter only partially surrounds the earth, and are both subject to the same disturbing influences of a difference of a temperature between the equator and the poles, it is reasonable to suppose that there is a similarity of their general motions. This is known from observation to be the case, except that the continents interfere more with the motion of the ocean than with those of the atmosphere. The general motion of the ocean in the torrid zone, where it is not interrupted by continents, is toward the west with an average velocity of about ten miles in twenty-four hours. Towards the poles the motion, in general, is towards the east, which is a necessary consequence of the preservation of areas; for if one part have a western motion, another part must have an eastern one, as was shown with regard to the atmosphere. If, then, there were no continents, there would be a general flowing of all the tropical parts of the ocean westward, and of the remaining parts toward the east. But when the tropical or equatorial current impinges against the eastern sides of the continents as in the Atlantic, a part is turned along the eastern side towards each pole. Likewise, when the eastern flow towards the poles, strikes against the western side of a continent, it is deflected towards the equator. Hence the northern parts of both the Atlantic and Pacific, have a tendency to a vortical motion, their tropical parts moving westward, and then turning northward on the eastern sides of the continents and joining the eastern flow, and south again towards the equator on the western sides of the continents. And it is evident from observation, that the southern parts of these oceans, and also the Indian ocean, have a tendency, in some measure, to the same kind of motions, except that the continents do not extend so far south, and consequently only a part of the eastern flow is turned toward the equator, the rest flowing on and producing the general eastern motion of the waters observed in the southern ocean.

The forces which produce the motions of the ocean.—The *primum mobile* of the motions of the ocean, as of the atmosphere, depends principally upon the difference of temperature between the equatorial and polar regions. The temperature of the ocean, on the surface at the equator, is about 80°, and it has a temperature above the mean temperature of the earth, which is 39°.5, to the depth of 7200 feet. Towards the poles it is below the freezing-point, and continues below the mean temperature at the parallel of 70°, to the depth of 4500 feet.* As water expands about 0.000455 of its bulk for every degree of increasing temperature, and sea-water contracts down to the temperature of 28°, calculations based upon these data, supposing the temperature to increase or decrease in proportion to the depth, make the specific gravity of the part at the equator, so much less than that at the poles, that it would have to rise about ten feet above the general level at the equator to be in equilibrium at the bottom of the sea, with the part at the poles. But then the equilibrium at the surface would be destroyed, and the waters would flow there towards the poles, where the superior pressure at the bottom over that of the equator, would cause a current to flow back at the bottom of the sea, towards the equator. Hence, if this cause of disturbance existed alone, there would be a current at the bottom of the sea from the poles towards the equator, moved by a force equal to the pressure of a stratum of water of about five feet, and one at the surface from the equator towards the poles, moved by an equal force. But this

* Mrs. Sommerville's Physical Geography, page 202.

motion, combined with the rotatory motion of the earth, gives rise to other forces, just as in the case of the atmosphere, which greatly modify these motions, as will be shown hereafter.

The preceding are the principal forces concerned in giving motion to the waters of the ocean. Lieutenant Maury, however, lays little stress upon these, and seems to think that the principal agencies concerned in these motions, arise from evaporation, the saltness of the ocean, galvanism, &c.* But we think it may be shown that these agencies can have no perceptible effect.

First, Lieutenant Maury supposes excessive evaporation to take place within the tropics and this vapor to be carried away and precipitated in extra-tropical regions, and infers that this would have, at least, a very sensible effect in producing the currents of the ocean. He puts the amount of evaporation of a stratum of one-half of an inch per day. Now if a stratum of water one-half of an inch in thickness is evaporated in twenty-four hours in one place and precipitated in another, it produces a difference of level of one inch between the two places, and the currents which it produces must be such as are sufficient to restore this level in the same space of time. Now we may judge how exceedingly small a current this would produce when we consider that there is a rise of about two feet in the open ocean at one place and a fall of the same amount at another every six hours, caused by the tides, and yet the flowing of the water from the one place to the other to produce this rise at one place, and fall at the other, it is well known does not produce any sensible currents in the open sea. Again, this matter can be easily reduced to calculation. If a stratum of water, one-half of an inch in thickness were taken up by evaporation from the torrid zone, and none of it precipitated there but all conveyed to the temperate and polar zones, it may be demonstrated upon the supposition that the ocean is four miles in depth, that the flow of water towards the equator to restore the equilibrium in the same time would not amount to a velocity of one foot per hour.

We think it may be likewise shown by calculations based upon reasonable hypothesis, if not entirely upon well-known data, that the salts of the sea also can have but little influence in producing currents. Lieutenant Maury makes a similar hypothesis in treating of the influence of the salt of the ocean, which he does in treating of the influence of evaporation, and supposes that the excess of salt left in the torrid zone by the excess of evaporation there, and the great precipitation in the temperate and polar regions produces such a difference in the specific gravity as to destroy the equilibrium of the sea, and to have a very sensible influence in producing currents, and especially the Gulf-stream.

With regard to the latter, he supposes that the water of the Gulf of Mexico has a much greater specific gravity than the water in the Atlantic, on account of the great evaporation to which it has been exposed in its passage from the coast of Africa across the Atlantic and through the Caribbean sea, and that, consequently, it is forced out into the Alantic by its greater pressure. Now, suppose it takes the water a year, which is about the actual time, to pass from the coast of Africa to the Gulf of Mexico; in this time, according to the hypothesis, there is evaporated a stratum of water fifteen feet in depth, and, as the salt contained in this stratum can not be evaporated, it remains in the part left, and increases its saltness. But sea-water contains only about three per cent. of saline matter, and consequently the amount of salt contained in this stratum of fifteen feet only increases the weight of the rest to an amount equal to the weight of a stratum of water about six inches deep. Hence, it only gives the water of the Gulf a tendency to flow out into the Atlantic with a force equal to the force with which a homogeneous fluid would flow out with its surface six inches above the general level of the Atlantic. This is much less than the opposing force arising from the great specific gravity of the water in the northern part of the Atlantic on account of its lower temperature, as we have shown by calculations. The same reasoning may be applied to any other part of the ocean; for, if the salt of the ocean has any influence in producing currents, it must be to produce an undercurrent from the torrid zone, where evaporation is supposed to be in excess, toward the poles, and, consequently, a counter-current at the surface from the poles toward the equator. But, upon any reasonable hypothesis, the water at the surface cannot lose by evaporation in passing from the poles to the equator, a stratum of water of such a depth, that the amount of salt contained in it can increase the specific gravity at the equator as much as the lower temperature increases it toward the poles; hence, if the salt of the sea has any sensible influence, it is only in opposition to a greater influence, and, consequently, it has a tendency to diminish, rather than increase, the currents of the ocean.

* See Physical Geography of the Sea §§ 26, 34 and 300.

We think it, therefore, manifest that neither evaporation nor the salts of the sea can have much influence in producing currents, even upon Lieutenant Maury's hopothesis, that evaporation is greatly in excess of precipitation in the torrid zone. But is this a true hypothesis? Although there is a great evaporation in the torrid zone, there is also great precipitation; for, with few exceptions, more rain falls at the equator than in any other part of the earth, and it is only the amount of evaporation over precipitation that should be taken into account, which we have reason to think is very small, and, if Professor Espy's theory is correct, it can not be anything; for, according to this theory, no vapor can pass from the torrid to the temperate zones and produce rain, since the current bearing it there would be a descending current, and consequently could not produce it.

The ocean not level.—As it has been shown in the case of the atmosphere, that the resultant of the forces causes an accumulation about the parallels of 28°, so, as the motions of the ocean are somewhat similar, and it is acted upon by the same forces, it may be shown that there must be a slight accumulation about those parallels in the ocean also. Whatever may be the causes of the motions of the ocean, we know that in the torrid zone it has a small western motion, and in the other parts a slight motion towards the east. The great equatorial current of the Atlantic moves about ten miles in twenty-four hours; but if we suppose that the average motion of the water in the torrid zone is five miles only per day, and that the maximum velocity of the water eastward in the extra-tropical regions is the same, using the same hypothesis which we did with regard to the atmosphere, the forces which result from these motions must cause an accumulation of more than forty feet about the parallels of 28°, above the level of the sea at the poles, and about five feet above the level of the equator This, however, would be the amount of accumulation to produce an equilibrium of the forces at the surface, but as this accumulation would then produce a greater pressure there upon the bottom than towards the poles and at the equator, it would produce, as in the case of the atmosphere, a flowing out from beneath this accumulation towards the poles and the equator, and settling down of the surface above, below the state of equilibrium, sufficient to cause a counter-current at the surface from the poles and the equater to supply the currents below. The accumulation there would be only about one-half that stated above, and there would be a flowing of water at the surface from both sides *towards* the parallels of 28°, and below a current in both directions *from* these parallels, similar to the motions of the atmosphere.

That the water of the ocean has such a motion as has been stated, appears from observations of its motions and other circumstances. Says Lieutenant Maury, "There seems to be a larger flow of polar waters into the Atlantic than of waters from it, and I cannot account for the preservation of the equilibrium of this ocean by any other hypothesis than that which calls in the aid of undercurrents." It is well-known that in Baffin's bay there is a strong surface-current running south, and a strong counter-current beneath running north. Another evidence of this general tendency of the waters, is, that icebergs, in both hemispheres, are drifted from the poles towards the equator, and in the south Atlantic sometimes reach the parallel of 37°. It is also evident from the fact that, in both the north Atlantic and Pacific oceans, there are large collections of drift and sea-weed about the parallels of 28°, so thickly matted that vessels are retarded in passing through them.*

These collections can only be formed by the flowing of the water at the surface from both sides to these parallels. It has been supposed that these collections are owing to the slight vortical motions of these oceans, it being supposed that any floating substances on the surface would have a tendency to collect at the vortex. This, however, would not be the case, for on account of friction at the bottom, the surface would have a greater vortical motion than the bottom, and consequently the water would be driven very slowly at the surface by the centrifugal force towards the sides, where it would cause a slight elevation and increase of pressure, which would cause the water to return towards the vortex at the bottom and not at the top; and hence floating substances at the surface could have no tendency to collect at the vortex.

We have corroborated these deductions from theory by numerous experiments made with a vessel of water with light substances on the surface. When the vessel is first receiving a vortical motion, the substances collect in the middle; for, as it is the vessel which gives motion to the water by means of the friction, the vessel, and consequently the bottom of the water, has then a greater motion than the top; and hence the reverse of what is stated above takes place; but, if the vessel is now stopped,

* Humboldt's Cosmos, vol. 2. page 278.

and the water within allowed to continue its motion, the vortical motion at the bottom is retarded faster than at the top, and soon has a slower motion there, when the light substances on the surface are seen to recede from the vortex towards the sides, and if there are any light substances on the bottom, they collect in the centre, all of which proves, that the water recedes from the middle at the surface, and returns to it at the bottom, and exactly agrees with the deductions from theory. These collections of sea-weed, then, cannot be caused by the vortical motions of the ocean, but must be the result of a general tendency of the surface water, to flow from both the equator and the poles towards these parallels; and, as it is prevented from collecting on these parallels near either side, on account of the slight vortical motion of the ocean, it collects only in the middle.

Explanation of the Gulf-stream.—We come now to the Gulf-stream, which has been a puzzle to philosophers ever since it was discovered. Many explanations have been given, and all known forces which can have any influence, have been brought in to account for this wonderful phenomenon. The most usual explanation is, that it is the escaping of the waters which have been forced into the Caribbean sea and the Gulf of Mexico by the trade-winds, which have been supposed to raise their surface above the general level, and thus afford a head as it were for the stream. This, without doubt, has a very considerable effect, but it has not generally been deemed adequate alone to account for the phenomenon, nor does it, in connection with all other known influences, afford a satisfactory explanation. "What is the cause of the Gulf-stream," says Lieutenant Maury, "has always puzzled philosophers. Modern investigation and examinations are beginning to throw some light upon the subject, though all is not yet clear."

We shall now endeavor to show that the additional force which we have taken into account in explaining both the winds and the currents of the ocean, and which seems to have been overlooked heretofore, will at least throw much additional light upon the subject, if not afford a complete explanation. We have shown that this force, which results from the eastward flow of the water in the extra-tropical regions, and from the western motion within the tropics, has a tendency to drive the water from the poles towards the equator, and also slightly from the equator towards the poles, and to produce an accumulation of at least twenty feet on the parallels of 28° above the level at the poles, upon the supposition that the maximum of this east and west flow is only five miles per day. But if, from any cause, the force which results from this eastward flow should be cut off at any place, the water would flow northward at that place with a force equal to that which would result from a head on the parallels of 28°, at least twenty feet above the level towards the poles. Now, it may be seen from the configuration of the coast of the United States, that this force is actually cut off along that coast; for this force depends upon the eastward flow of the water there, which it cannot have, inasmuch as it must flow in both ways along the coast to fill up the vacuum which such a motion would produce. As the Gulf of Mexico, therefore, and the adjacent parts of the Atlantic, lie in the parallel of greatest accumulation, the water must flow from these parts along the coast with a force equal to that stated above. In addition to this, the momentum of the water flowing westward in the torrid zone, with a motion depending upon the prime moving course, due to a difference of specific gravity between the poles and the equator, in connection with the rotatory motion of the earth, and being independent of the effect of the trade-winds, must force the water in the Caribbean sea and the Gulf of Mexico considerably above the general level and add to the preceding force. When we consider that the motion of the water which produces tides on our coasts, is in general imperceptible in the open ocean, and yet, on account of the sloping bottom of the ocean, which causes a smaller volume of water to receive the momentum of a larger one, it causes considerable rise of the water along the coast, we have reason to think that the general tendency of water westward in the torrid zone may keep the water in the Gulf considerably above the general level, since its water and that in the Caribbean sea, if the bottom of the ocean be sloping, must in a great measure receive the momentum of the whole body of the water moving westward in the adjacent part of the Atlantic. The eastern tendency of the water in the northern part of the north Atlantic, due to the prime moving force mentioned above and independent of the winds which prevail there, causes the surface of the ocean in the latitude of Newfoundland to be somewhat depressed below the general level next to the coast, which also adds to the force of the Gulf-stream. All these forces, taken in connection with the influence of the trade-winds, to which this phenomenon has been mainly attributed, we think, furnish a complete and satisfactory explanation of that great wonder and mystery of the ocean, the Gulf-stream.

The Greenland and other currents.—The general eastward motion of the waters of the ocean in the

northern part of the Atlantic, and consequent depression next the coast of North America, also furnish an explanation of the cold current of water flowing between the Gulf-stream and the coast of the United States, called the Greenland current. On account of the rotatory motion of the earth, the water of the Gulf-stream in flowing northward, tends to the east, and for the same reason the water flowing from Greenland and Baffin's bay to supply the eastern flow, tends towards the west, and consequently flows in between the Gulf-stream and the coast of the United States.

There must be a motion of the waters somewhat similar to that of the Gulf-stream and the Greenland current, wherever the great equatorial current impinges against a continent, and the eastward flow towards the poles is cut off. Hence, on the eastern coast of South America, there is a warm Brazilian current towards Cape Horn, and on the eastern coast of Africa, the Mozambique current which at the Cape of Good Hope is called the Agulhas current. Also, on the eastern coast of Asia, there is the warm China current, flowing towards the north, similar to the Gulf-stream, and the cold Asiatic current, insinuating itself between it and the coast, like the Greenland current.

On the western side of the continents a motion somewhat the reverse of this must take place. Hence, instead of a warm stream flowing towards the north, there is a cold current flowing towards the equator. On the west of Portugal, and the northern part of Africa, there is a flow of colder water towards the equator, and west of the southern part of Africa, is the cold Atlantic current, also tending towards the equator, both to join the great equatorial current flowing across the Atlantic. On the west coast of North America, also, there is a flow of colder water along the coast from the north, and on the west coast of South America, is Humboldt's current, 8° or 10° colder than the rest of the ocean in the same latitude, both tending towards the equator to there join the great western current across the Pacific, and to fill up, as it were, the vacuum which this current has a tendency to leave about the equator, on the western coast of America.

NASHVILLE, *October* 4, 1856.

NO. 2.

THE MOTIONS OF FLUIDS AND SOLIDS RELATIVE TO THE EARTH'S SURFACE.

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1. Under the above title I have published a paper in the "Mathematical Monthly," in which I endeavored, by a complete analytical investigation of the motions of fluids surrounding the earth, arising from the different circumstances of a difference of pressure caused by a difference of temperature, combined with the modifying forces arising from the earth's rotation, to give a satisfactory explanation of all the general motions of the atmosphere and the ocean, the cause of the greater barometric pressure of the atmosphere near the tropics than at the equator and the poles, and of the greater pressure generally in the northern hemisphere than in the southern, to account for the motions of revolving storms in both hemispheres, from the equator towards the poles in the parabolic paths, and to establish completely their gyratory character; none of which phenomena had ever been satisfactorily accounted for by any of the theories which do not take into account the effect of the earth's rotation. It has been suggested by some that a paper more popular in its character, although less complete, which should contain only the more essential parts of the analysis, and in which familiar illustrations should supply in some measure the more difficult parts of the analysis, would be more satisfactory to many readers. It is proposed, therefore, in this paper, in consequence of the general interest taken in the subject, to treat it according to this suggestion, and to give only the most essential part of the analysis, showing the influence of the earth's rotation, which being based upon well-known principles, instead of being deduced from general fundamental equations, is very simple, but is sufficient for a general understanding of the subject.

I. *The Effect of the Earth's Rotation upon Moving Bodies at its Surface.*

2. If a body were set in motion upon the surface of the earth, supposed to be entirely without friction, it would not in general move in the circumference of the great circle around the earth, but would be continually deflected to one side by a force arising from the earth's rotation.

Let r be the radius of the earth,
θ the polar distance in arc,
φ the longitude,
n the angular velocity of the earth's rotation.

Then $r \sin \theta$ is the distance of a body on the earth's surface from the axis of rotation, and $r \sin \theta\, n^2$ the centrifugal force in the direction of a perpendicular to the earth's axis, arising from the earth's rotation. If, in addition to the angular motion common to all bodies at rest on the surface of the earth, the body has an angular motion $D_t \varphi$ relative to the earth, then the centrifugal force becomes $r \sin \theta\, (n + D_t \varphi)^2$. Now if we resolve the preceding force in the directions of the meridian and a perpendicular to it, the part acting in the direction of the meridian, neglecting the small effect of the earth's ellipticity, is $r \sin \theta \cos \theta\, (n + D_t \varphi)^2$. The part of this force which gives ellipticity to the earth's surface, and which is necessary to keep a body at rest on the elliptical surface, and prevent it from sliding toward the pole, is $r \sin \theta \cos \theta\, n^2$. Hence the difference of these two forces, when the body has a motion eastward relative to the earth, is a deflecting force which has a tendency to cause the body to move from the pole to the equator. The difference of these forces is $r \sin \theta \cos \theta\, (2n + D_t \varphi)\, D_t \varphi$, and hence when the body is entirely free to move in any direction, we have

$$r\, D_t^2\, \theta = r \sin \theta \cos \theta\, (2n + D_t \varphi)\, D_t \varphi. \qquad (1)$$

3. Again, if the body has a motion toward or from the pole, it must satisfy the well-known principle of the preservation of areas, so that as it approaches the pole, and consequently the axis of rota-

tion, the angular motion must be increased, that is, it must acquire a motion eastward relative to the earth, but if it recedes from the pole, it must acquire a relative westward motion. In order to satisfy the preceding principle, the motion must satisfy the following equation:

$$r^2 \sin^2 \theta \, (n + D_t \varphi) = \text{constant}. \tag{2.}$$

Taking the derivative with regard to t we get

$$r \sin \theta \, D_t^2 \varphi = -2 r \cos \theta \, (n + D_t \varphi) \, D_t \theta. \tag{3.}$$

4. Equations (1) and (3) determine the motions of a free body on the earth's surface. If the body is constrained to move either in the direction of a meridian or a parallel of latitude only, the deflecting force instead of causing a deflection, causes only a pressure. If we put P for the part of the force or pressure acting in the direction of the meridian which depends upon the earth's rotation only, and Q for the part depending upon the earth's rotation which acts in the direction of a parallel of latitude, we have

$$\begin{cases} P = 2 r n \sin \theta \cos \theta \, D_t \varphi \\ Q = -2 r n \cos \theta \, D_t \theta. \end{cases} \tag{4.}$$

In the preceding equations $r D_t \theta$ represent the lineal velocity of the body in the direction of the meridian, and $r \sin. \theta D_t \varphi$ the lineal velocity relative to the earth in the direction of the parallel of latitude. Hence the deflecting force is the same in both directions in the same lineal velocity.

5. If v is the velocity of a body moving in any direction whatever, and F the deflecting force perpendicular to this direction, by resolving the preceding forces and velocities in the direction of v and the perpendicular to it on the right we get

$$F = 2 n v \cos \theta. \tag{5.}$$

In the northern hemisphere, cos. θ is positive, but in the southern, negative. Hence, we have established this important principle, *in whatever direction a body moves, it is always deflected to the right in the northern hemisphere, and the contrary in the southern hemisphere.*

The forces resolved in the direction of v cancel each other, and hence the velocity is never accelerated or retarded.

6. Since $r n^2$ represents the centrifugal force at the equator, arising from the earth's rotation, and is known to be $\frac{1}{289}$ of g or gravity, the preceding equation may be reduced to

$$F = \frac{2 v \cos \theta}{289 n} g. \tag{6.}$$

This form of the equation is convenient for comparing the force F with gravity.

7. In the preceding equation $r n$ is the lineal velocity at the equator, and is equal to 1523.2 feet, the second being the unit of time. Hence $n = 000072924$. Also $g = 32.2$ feet.

It may be remarked here that the preceding deflecting force is not an absolute force, such as would be required to deflect a moving body from a fixed direction in space, but is only relative, being somewhat of the nature of a centrifugal force, and arises from the fact that the direction relative to the earth to which the moving body is referred, is continually changing its direction with regard to fixed directions in space.

II. *The General Motions and Pressure of the Atmosphere.*

8. By the general motions of the atmosphere are meant all those motions produced by a difference of density between the equatorial and polar regions arising principally from a difference of temperature. If the atmosphere in all parts of the earth had the same density, every part would be in a state of statical equilibrium, and its surface and the strata of equal density would assume the elliptical figure of the earth's surface, and consequently the pressure of the atmosphere at the earth's surface would be everywhere the same. But the temperature of the atmosphere being less, and consequently its density greater, in the polar than the equatorial regions, the greater pressure of the polar regions causes the surfaces of the atmosphere and the strata of equal density in the equatorial regions to rise a little above the level of equilibrium, and hence the atmosphere in the upper regions flows toward the poles, while the greater pressure of the polar regions causes a counter-current toward the equator in the parts nearer the earth's surface, which would extend down to the earth's surface, if it were not for the modifying causes arising from the friction of the earth's surface, which will be explained. If the earth had no rotation on its axis, this interchanging motion between the equatorial and polar regions would be exactly in the directions of the meridians; but having a rotation, from what has been demonstrated in the preceding section, the atmosphere above in flowing toward the poles acquires an eastward motion relative to the earth's surface, and after decending in

the polar regions and flowing back nearer the earth's surface toward the equator, it tends toward the west, and on arriving in the equatorial regions it has a westward motion. If it were not for the resistance of the earth's surface, the mutual actions of the strata upon one another, whatever the initial state of the atmosphere, would cause them to have finally the same east or west motion at all heights in the same latitude, and this motion would be such as to satisfy equation (2), and also, since the mutal actions of the strata upon one another could not effect the sum of the moments, it would be such that the sum of the moments of all the particles would be the same as that of the initial state arising from the earth's rotation and from any initial motion relative to the earth, which it might have had. This latter condition determines the constant in equation (2), which was shown in my paper in the "Mathematical Monthly," to which I must refer for the method, to be, on the hypothesis of an inital state of rest relative to the earth, $\frac{2}{3} r^2 n$. With this value of the constant the equation gives

$$D_t \varphi = \left(\frac{2}{3 \sin^2 \theta} - 1 \right) n. \tag{7.}$$

9. Near the poles, where $\sin^2 \theta$ is very small, $D_t \varphi$ and consequently the lineal velocity $r \sin \theta \, D_t \varphi$ must be very great. Toward the equator $D_t \varphi$ is negative, and hence the motion there is westward. Putting $D_t \varphi = 0$, the equation gives $\sin^2 \theta = \frac{2}{3}$, and hence the sine of the latitude where there is no motion east or west is equal to $\sqrt{\frac{2}{3}}$, which answers nearly to the parallel of 35°. Hence between this parallel and the poles the motion is eastward, but between it and the equator, toward the west. From what has been shown therefore in section 2, the eastward motion toward the poles, and the westward toward the equator, since $D_t \varphi$ is negative there, must both cause a pressure toward the parallels of 35°, and consequently an accumulation of atmosphere there, and a depression at the poles and at the equator. The amount of the pressure is represented by the term $r \sin \theta \cos \theta \, (2n + D_t \theta) \, D_t \varphi$, and hence toward the pole, where $D_t \varphi$ is very large, this pressure is very great, and at the poles it would be infinite. Hence at the poles and at some distance from them, on the hypothesis of no resistances from the earth's surface, the atmosphere cannot exist. Between the parallels of 35° and the equator $D_t \varphi$ is less and also $\cos \theta$, and consequently the pressure from the equator and the depression there are comparatively small. It was shown in the "Mathematical Monthly" that in the case of a fluid surrounding the earth five miles high, the fluid would recede from the poles about 28° and be depressed at the equator about 4,000 feet. This, however, is upon the hypothesis that the upward expansion of the atmosphere arising from a greater temperature is insensible.

10. The preceding results, it must be remembered, are all upon the hypothesis that the atmosphere is not resisted in its motions by the friction of the earths's surface. Although these results are much modified by the resistance of the earth's surface, yet they will be of great advantage in explaining its general motions; for, as there can be no resistance until there is motion, the atmosphere must have a tendency to assume, in some measure, the same motions and figure as in the case of no resistances. Hence, towards the poles the general motions of the atmosphere must be towards the east, and in the torrid zone towards the west, but as these motions, in consequence of the resistances, are small in comparison with those in the case of no resistance, instead of the atmosphere's receding entirely from the poles, there must be only a comparatively small depression there, as represented in the figure, and instead of its being about 4,000 feet lower at the equator than at the place of its maximum height near the tropics, there must be only a very slight depression there, both on account of the small pressure from the equator, and also on account of the upward expansion arising from a greater temperature.

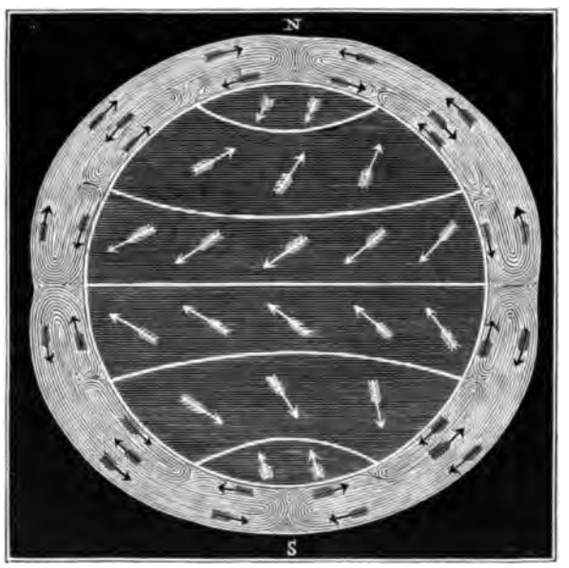

Figure 1.

11. That the atmosphere must assume the preceding figure in consequence of the eastward motion toward the poles and westward motion near the equator, will be readily understood from the following

illustration: It is well known that if the atmosphere has the same motion of rotation with the earth, that it would assume the same figure. Now, if the whole atmosphere had a greater angular motion, that is, if it had an eastward motion relative to the earth, the increased centrifugal force would evidently cause it to accumulate at the equator and to be depressed at the poles. On the other hand, if it had a less angular motion, that is, a motion westward relative to the earth, it would accumulate at the poles and be depressed at the equator. Hence, since the motion is eastward in the higher latitudes and westward near the equator, it is plain that there must be a depression at both the poles and the equator, and consequently an accumulation having its maximum height at some parallel between.

12. The force which overcomes the resistance of the earth's surface to the east and west motion of the atmosphere depends upon the term in equation (3) containing $D_t\,\theta$ as a factor, which depends upon the interchanging motion of the fluid between the equatorial and polar regions, and hence the term must vanish at the equator and the poles. All the east or west motion of the atmosphere is consequently destroyed by the resistances at these places, and hence, as $D_t\,\theta$ vanishes there also, there is a belt of calms at the equator, called the equatorial calm-belt, and there must be also, a region of calms about the poles.

13. As the motion of the atmosphere is east towards the poles and west near the equator, somewhere between the equator and the poles there must be a parallel of no motion east or west, which, in the case of no resistance, was determined upon the hypothesis of an initial state of rest, and found to be at the parallel of 35°, (section 9.) In the case of the atmosphere, this parallel is entirely independent of the initial state of the atmosphere, and depends, in a great measure, upon the law of resistance, and hence it cannot be accurately determined. It is evident, however, that the east and west motions of the atmosphere at the earth's surface must be such that the sum of the resistances of each part of the earth's surface multiplied into its distance from the axis of rotation, must be equal to 0, else the velocity of the earth's rotation would be continually accelerated or retarded, which cannot arise from any mutual action between the surface of the earth and the surrounding atmosphere. Now, as the part of the earth's surface where the motion of the atmosphere is west is much farther from the axis than the part where it is east, the latter part must comprise more than half of the earth's surface, unless the velocity of the eastern motion towards the poles is much greater than that of the western motion near the equator. Therefore, since one-half of the earth's surface is contained between the parallels of 30°, the parallels of no east or west motion at the earth's surface must fall within these parallels, and they are accordingly found on the ocean to be near the tropics. Hence the maximum height of the atmosphere, as represented in the figure, must also be near the same parallels.

14. The increase of pressure arising from the accumulation of atmosphere near the tropics, caused, principally, by the deflecting forces (section 5) arising from the more rapid east and west motions of the atmosphere in the upper regions, where there is least resistance, gives the atmosphere a tendency to flow from beneath this accumulation both towards the equator and the poles, since the motions, and consequently the forces which cause this accumulation, are much less near the surface; but, on account of the greater density of the atmosphere towards the poles it has a tendency also to flow, at the earth's surface, from the poles towards the equator. Between the parallels of greatest pressure and the equator these tendencies combine and produce a strong surface current, which, combining with the westward motion there, gives rise to the well-known northeast wind in the northern hemisphere and the southeast wind in the southern hemisphere, called the trade-winds; but between the parallels of greatest pressure and the poles these tendencies are opposed to each other, and the one arising from the accumulation of atmosphere near the tropics, being the greater in the middle latitudes, causes the atmosphere to flow at the earth's surface towards the poles, and this motion, combining with the general eastward motion of the atmosphere in those latitudes, gives rise to the southwest wind in the northern hemisphere and the northwest wind in the southern hemisphere, called the passage-winds.

15. Near the poles the tendency to flow towards the equator seems to be greater, and causes a current there *from* the poles, which, being deflected westward (section 5), causes a slight northeast wind in the north frigid zone and a southeast wind in the south frigid zone; but this is only near the earth's surface, and the general tendency of the atmosphere in the upper regions must be towards the east, as will be seen.

16. Since the atmosphere near the tropics can have no motion in any direction at the earth's surface, there are calm-belts there, called the tropical calm-belts. Near the polar circles, where the

polar and passage-winds meet, there must also be calm-belts, which may be called polar calm-belts. The motions of the atmosphere, therefore, at the earth's surface, if they were not modified by the influence of continents, would be as represented in the interior of the figure, in which the heavy lines represent the calm-belts. On account of the influence of the continents, these belts are somewhat displaced and irregular, and on account of the varying position of the sun, they change their positions a little in different seasons of the years. The southern limit of the polar winds in the northern hemisphere, and also the limit between the trade and passage-winds, has been determined by Professor J. H. Coffin, from the discussion of a great number of observations at different points, and given in a chart, in his treatise of the winds, published in the seventh volume of the "Smithsonian Contributions."

17. That the atmosphere is depressed at the equator and the poles, and has its maximum height near the tropics, as has been represented, is indicated by barometrical pressure. It was formerly thought that this pressure at the level of the ocean, was very nearly 30 inches in all latitudes; but it is now well established that it is much less towards the poles than near the tropics, and also a little less at the equator. Says Captain Wilkes: "The most remarkable phenomenon which our observations have shown is the irregular outline of the atmosphere surrounding the earth as indicated by the pressure upon the measured column at different parts of the surface. Our barometrical observations show a depression within the tropics, a bulging in the temperate zone, again undergoing a depression on advancing towards the arctic and antarctic circles."

Says Sir James Ross: * "Our barometrical experiments appear to prove that the atmospheric pressure is considerably less at the equator than near the tropics; and to the south of the tropic of Capricorn, where it is greatest, a gradual diminution occurs as the latitude is increased, as will be shown from the following table, derived from the hourly observations of the height of the column of mercury between the 20th of November, 1839, and the 31st of July, 1843."

Extract from Ross's Table.

Latitude.	Pressure.	Latitude.	Pressure.	Latitude.	Pressure.
	Inches.		*Inches.*		*Inches.*
Equator	29.974	42° 53′	29.950	55° 52′	29.360
13° 6′ S.....	30.016	45 00	29.664	60 00	29.114
22 17	30.085	49 08	29.467	66 00	29.078
34 48	30.023	51 23	29.497	47 00	28.928
		54 26	29.347	74 00	

18. The following table just published by M. Schouw, and reduced here from millimeters to English inches, shows that there is a bulging of the atmosphere in the middle latitudes and depression at the poles in the northern hemisphere, similar to that observed in the southern hemisphere.

Place.	Latitude.	Pressure.	Place.	Latitude.	Pressure.
		Inches.			*Inches.*
Cape	33° 0′ S.	30.040	London.....	51° 30′	29.961
Rio Janeiro	23 S.	30.073	Altona	53 30	29.937
Christianburg.....	5 30 N.	29.925	Dantzic.....	54 30	29.925
La Guayra	10	29.928	Konigsburg	54 30	29.941
St. Thomas.....	19	29.941	Apenrade	55	29.905
Macao.....	23	30.039	Edinburg.....	56	29.851
Teneriffe	28	30.087	Christiania.	60	29.866
Madeira.....	32 30	30.126	Bergen.....	60	29.703
Tripoli.....	33	30.213	Hardanger...	60	29.700
Palermo.....	38	30.036	Reikiavik	64	29.607
Naples	41	30.012	Godthaab	64	29.603
Florence	43 30	29.996	Eyafiord.....	66	29.669
Avignon	44	30.000	Godhavn..	69	29.674
Bologna.....	44 30	30.008	Upernavik	73	29.732
Padua.....	45	30.008	Mellville Isle.....	74 30	29.807
Paris.....	49	29.976	Spitzbergen.....	75 30	29.795

19. From the preceding tables it is seen that the barometric pressure is much less, especially in the southern hemisphere, towards the poles than at the equator, although the density toward the poles is much greater, and hence the depression there must be considerable.

20. It has been seen (section 1), that, in consequence of the earth's rotation, the interchanging motion of the atmosphere between the equator and the poles gives rise to a force, by which this motion itself is counteracted. For instance, the motion toward the poles in the upper regions causes an eastward motion which gives rise to a force toward the equator, and which consequently counteracts the

* Voyage to the southern seas, vol. ii., p. 383.

motion toward the poles, and the motion towards the equator produces a westward motion which gives rise to a force acting in the direction of the poles, which counteracts the motion toward the equator. The motion of the atmosphere, therefore, between the equator and the poles, is not produced by the whole force arising from the difference of density between the equator and the poles, but by a small part only of this force. Hence, if the earth had no rotatory motion, the force which produces this motion would be very much greater, and there would be a sweeping hurricane from the pole to the equator.

21. It is evident, where the motions of the atmosphere are resisted by the earth's surface, that all the conditions cannot be satisfied by a motion at the surface from the poles towards the equator, and by a counter-motion in the upper regions. For we have seen (section 13), that the atmosphere at the surface of the earth must have an eastern motion in the middle latitudes; but it cannot have such a motion, unless it also has a motion toward the poles, in order that the deflecting force (section 5) arising from this motion may overcome the resistance to the eastern motion. But it is evident there cannot be a complete reversal of the motions in the middle latitudes, but some portion of it must flow toward the poles in the upper regions, else the eastern motion there could not be greater than at the surface, which the conditions require. The motions, therefore, must be somewhat as represented in the figure. The part of the atmosphere next the earth's surface in the middle latitudes having a motion toward the poles, extends to a considerable height, since it generally embraces the region of fair-weather clouds, as may be seen by observation.

22. Since the force arising from the eastward motion of the atmosphere above must resist in a great measure the tendency to flow from the equator to the poles, caused by the difference of level arising from a greater upward expansion of the atmosphere near the equator on account of a greater temperature, and since this difference of level, and consequently the tendency to flow toward the poles, must increase with the height, the eastward motion in the middle and higher latitudes must be much greater above than below, and in the trade-wind region, near the tropical calm-belts, where the motion is westward below it must be toward east above. This is also evident from the general consideration, that the whole amount of deflecting force eastward arising from the motion of the atmosphere towards the poles, is equal to the deflecting force westward arising from its motion back towards the equator, and that the deflecting force eastward is principally above where there is less resistance than near the surface. Hence, at the top of Mauna Loa in the Sandwich Islands, and on the peak of Teneriffe, both of which places are near the tropical calm-belt at the surface, a strong southwest wind prevails. Hence, also, "on the eruption of Saint Vincent, in 1812, ashes were deposited at Barbadoes, sixty or seventy miles eastward, and also on the decks of vessels one hundred miles still further east, while the trade-wind at the surface was blowing in its usual direction." The eastward motion of the atmosphere above, in the latitudes of the trade-winds, is also confirmed by the observations made on the directions of the clouds at Colonia Tovar, Venezuela, latitude 10° 26′, as given in the Report of the Smithsonian Institution for 1857 (page 254). While the motion of the lower clouds was in general from some point toward the east, the observed motion of nearly all the higher clouds was from some point toward the west.

23. From what precedes, the limit between the atmosphere which moves eastward in the middle latitudes and westward nearer the equator, which at the earth's surface is at the tropical calm-belt, must be a plane inclining toward the equator above. And since, according to section 21, the atmosphere near the earth's surface cannot have an eastward motion, unless it also has a motion toward the poles, this plane near the earth's surface must nearly coincide with the one which separates the atmosphere moving towards the poles, from that moving towards the equator, in the trade-wind regions, and hence the latter must also incline above towards the equator. This explains the winds at the peak of Teneriffe, which at the top always blow from the southwest, while at the base they blow alternately from the northwest and northeast, changing with the seasons. As the tropical calm-belt together with this dividing plane changes its position with the seasons, as will be explained when this plane, in the latter part of summer, is farthest north, it still leaves the top of the peak north of it while the base is south of it; and hence the wind at the top always blows from the southwest, even when at the base it blows from the northeast. As this plane moves south in the fall, more of the peak gradually becomes north of it, and hence the southwest wind, which always prevails at the top, gradually descends lower on the sides of the peak until it reaches the base. Hence, when this plane reaches its most southern position, in the latter part of the winter, the southwest wind prevails at both the base and the top.

24. The depression of the atmosphere at the poles and at the equator, and the accumulation near the tropics, may be explained in a general manner by means of the principle in (section 5) that when a body moves in any direction in the northern hemisphere it is deflected to the right, and the contrary in the southern. The atmosphere towards the poles having an eastward motion, the deflecting force arising from it causes a pressure towards the equator, and the motion near the equator being westward, the pressure is towards the poles; and hence there must be a depression at the poles and at the equator, and an accumulation near the tropics. Since this deflecting force is as $\cos \theta$, it is small near the equator, and consequently the depression there is small.

25. According to the preceding tables of barometric pressure, there is more atmosphere in the northern than in the southern hemisphere. Says Sir James Ross, "The cause of the atmosphere being so very much less in the southern than in the northern hemisphere remains to be determined." This is very satisfactorily accounted for by the preceding principle, for, as there is much more land with high mountain ranges in the northern hemisphere than in the southern, the resistances are greater, and consequently the eastern motions, upon which the deflecting force depends, is much less; and the consequence is, that the more rapid motions of the southern hemisphere cause a greater depression there, and a part of the atmosphere to be thrown into the northern hemisphere.

This also accounts for the mean position of the equatorial calm-belt being, in general, a little north of the equator; but in the Pacific ocean, where there is nearly as much water north of the equator as south, its position nearly coincides with the equator.

For the same reason the tropical calm-belt of the northern hemisphere is farther from the equator than that of the southern hemisphere, and, on account of the irregular distribution of the land and water of the two hemispheres in different longitudes, it does not coincide with any parallel of latitude. In the longitude of Asia, where there is land only in the northern hemisphere and the Indian ocean in the southern, this belt, which is also the dividing line which separates the winds which blow east from those which blow west, is farther from the equator than at any other place, as shown by Professor Coffin's chart.

26. In winter, the difference of temperature between the equator and the poles, upon which the disturbance of the atmosphere depends, is much greater than in the summer. This causes the eastward motion of the atmosphere in either hemisphere during its winter to be greater, while in the other hemisphere it is less. Hence, a portion of the volume of the atmosphere in winter is thrown into the other hemisphere; but, although the volume or height of the atmosphere is then less, yet, being more dense, the barometric pressure remains nearly the same. The difference at Paris, and in the middle latitudes generally, between winter and summer, is only about $\frac{1}{10}$ of an inch.

On account of this alternate change with the seasons of the velocity of the eastward motion of the atmosphere in the two hemispheres, the equatorial and tropical calm-belts change their positions a little, moving north during the spring and south in the fall.

III. *The Motion of the Atmosphere arising from local disturbances.*

27. Besides the general disturbance of equilibrium arising from a difference of specific gravity between the equator and the poles, which causes the general motions of the atmosphere, treated in the last section, there are also more local disturbances, arising from a greater rarefaction of the atmosphere over limited portions of the earth's surface, which give rise to the various irregularities in its motions, including cyclones or revolving storms, tornadoes, and water-spouts. When, on account of greater heat, or a greater amount of aqueous vapor, the atmosphere at any place becomes more rare than the surrounding portion, it ascends, and the surrounding heavier atmosphere flows in below, to supply its place, while a counter-current is consequently produced above. As the lower strata of atmosphere generally contain a certain quantity of aqueous vapor, which is condensed after arising to a certain height, and forms clouds and rain, the caloric given out in the condensation, in accordance with Espy's theory, produces a still greater rarefaction, and doubtless adds very much to the disturbance of equilibrium, and to the motive power of storms. So long, then, as the ascending atmosphere over the area of greater rarefaction is supplied with aqueous vapor by the current flowing in from all sides below, the disturbance of equilibrium must continue, and consequently the local disturbances of the atmosphere to which it gives rise, whether those of an ordinary rain storm, or a cyclone, may continue many days, while the general motions of the atmosphere may carry this disturbed area several thousands of miles.

28. When the area of rarefaction is such as to cause the atmosphere to flow in below from all sides toward a centre and the reverse above, thus enabling a constantly interchanging motion between the internal and external part, the case becomes very similar to that of the general hemispherical motions of the atmosphere in which the motion is between the polar and equatorial parts. For if the earth's rotation on its axis is analyzed with reference to any other axis, the pole of which is at the distance of θ from the pole of the earth, it is found to have a rotation around this latter axis equal to $n \cos \theta$ (Peirce's "Analytical Mechanics," § 25). Hence the interchanging motion between the internal and external part in this case must cause the internal part to gyrate around the centre from right to left in the northern hemisphere and the external part the contrary way, and thus give rise to a cyclone or revolving storm, just as in the case of the hemispherical motions the part nearest the pole acquires an eastward motion, and that near the equator a westward motion. This is also evident from the principle demonstrated in section 5, according to which the atmosphere in flowing toward the centre below, must be always deflected in the northern hemisphere to the right, and consequently give it a gyratory motion around the centre from right to left. In moving out above toward the external part, the deflection tends to give the atmosphere a gyratory motion the contrary way, and hence, in flowing out above, the gyratory motion which it has while rising to the upper strata in the interior, is not only destroyed but, on arriving at the external part, it has a gyratory motion the contrary way. In the southern hemisphere, since the deflections there are all to the left of the direction of motion, the gyrations are all reversed, which is the observed law of storms in all parts of the world, as shown by Redfield, and also by Reid in his "Law of Storms." It is also evident that at the equator, where $\cos \theta$, upon which the deflecting force depends, vanishes, there cannot be a cyclone, and hence, of all those which Redfield has investigated and given in his charts of their routes, none have been traced within 10° of the equator. The typhoons or cyclones, also, of the China sea, have never been observed within 9° of the equator.

29. In the case of the general hemispherical motions the internal or polar part is most dense, but in the case of cyclones, the external part. Hence the motions between the centre and external part are different; in the former case the motion toward the centre being above, but in the latter below. In the case of no resistances from the earth's surface, this does not affect the gyrations, since, on account of the actions of the different strata upon one another, they all eventually have the same gyratory motion. But where there are resistances, they cause gyrations in the external part to be the most rapid above, just as in the system of hemispherical motions, they are most rapid above in the internal or polar part.

30. If the gyrations were not resisted by the earth's surface, it is evident that the rapid gyrations of the cyclone would cause the atmosphere to recede entirely from the centre, but on account of the resistance the very rapid gyrations are in general in a great measure prevented, so that instead of a complete vacuum, the strata of the atmosphere are only somewhat depressed in the interior, as represented in figure 2. For since the force which produces the gyrations depends upon the velocity of the flow to and from the centre, it is evident that, at the centre and at the external part of the disturbed portion of atmosphere, where this velocity must vanish, the resistances destroy all gyratory motion. Hence, instead of very rapid gyrations near the centre, as in the case of no resistances, there must be a calm there, and the most rapid gyrations be at some distance from the centre, in accordance with observation. The diameter of the comparatively calm portion, in the centre of the large cyclones, is sometimes about thirty miles. The velocity of gyration of the external part, which in the case of no resistances, is small, is in a great measure destroyed by the resistances of the surrounding atmosphere, so that it is, for the most part, insensible to observation, and only the more rapid gyrations of the internal part are observed. This motion of gyration combined with the motion at the earth's surface towards the centre, gives rise to a spiral motion in the interior, exactly in accordance with the observed motions of the atmosphere in great storms or hurricanes, as has been shown by Redfield, in a number of papers on the subject, published in this journal. [American Journal of Science.]

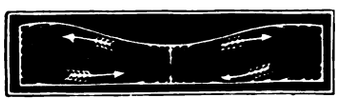

Figure 2.

31. Since the atmosphere is depressed in the middle of cyclones, they must sensibly affect the barometer; and this is the true cause of all great barometrical oscillations, as was first suggested by Redfield*. As the cyclone approaches, there is generally a very slight rise of the barometrical column,

* This journal, [American Journal of Science,] [1] xx, 45.

which is at its maximum at the greatest accumulation near the external part of the cyclone, after which it is gradually depressed until the middle of the cyclone arrives, where the atmosphere is most depressed, when the barometer is at its minimum, and then it returns in a reverse manner to its former height, when the cyclone has passed. In great storms, the mercury sometimes falls more than two inches. In oblong storms, and all imperfectly developed cyclones, the same phenomena must take place in some measure, as in a complete cyclone. We have reason to conclude, therefore, that nearly all the oscillations of the barometer are caused by a cyclonic motion of the atmosphere, by which it is depressed in the middle of the cyclones. The cyclones may be very irregular and imperfectly developed, and not of sufficient violence to produce a strong wind, and several may frequently interfere with one another, so that the oscillations may frequently be very slight ones only and very irregular.

Since the gyratory motion of a cyclone, and the consequent depression at the centre, depend upon a term containing as a factor, $\cos \theta$, (section 28), which is the sine of the latitude, the oscillations, according to the preceding theory of barometrical oscillation, should be small near the equator, and increase towards the poles, somewhat as the sine of the latitude. Accordingly, at the equator, the mean monthly range of oscillation is only two millimeters, or less than $\frac{1}{10}$ of an inch, while there is a gradual increase with the latitude; so that at Paris it is 23.66^{mm}, and at Iceland, 35.91^{mm}. (Kaemtz's Meteorology, by C. Walker, page 297.)

32. The greater rarefaction of the atmosphere at some times than at others, without doubt, has considerable effect upon the barometer; but the theory which attributes the whole of the barometrical oscillations to the rarefaction of the atmosphere produced by the condensation of vapor in the formation of clouds and rain, cannot be maintained; for according to that theory, in the rainy belt near the equator, where there are always copious rains during the day, which are succeeded by clear atmosphere during the night, the oscillations of the barometer should be the greatest, and towards the poles, where there is little condensation of vapor into rain, they should be the least, but we have seen that just the reverse of this is true. The atmosphere is extremely mobile, so that if there were much difference of pressure between one part and the surrounding parts, the equilibrium would be at once restored if the motions toward the centre produced by this difference of pressure, did not give rise to a cyclonic motion, and thus to a centrifugal force combined with the deflecting force arising from the earth's rotation, which in a great measure counteracts the force arising from a difference of pressure between the external and internal parts. If, therefore, the earth had no rotation, the oscillations of the barometer would be no greater in any part of the earth than they are at the equator.

33. When the disturbance of equilibrium is great, but extends over a small area only, the centripetal force is much greater than in the case of large cyclones, and the gyrations are then very rapid and very near the centre, as in the case of tornadoes. Tornadoes generally occur when the surface of the earth is very warm and the atmosphere calm. For then the strata near the surface becomes very much rarefied, and are consequently in a kind of unstable equilibrium for a while, when from some slight cause, the rarefied atmosphere rushes up at some point through the strata above, and consequently flows in rapidly from all sides below, and then, unless the sum of all the inital moments of gyration around the centre is exactly equal 0, which can rarely ever be the case, it must run into rapid gyrations near the centre, and a tornado is the consequence. This may be exemplified by the flowing of water through a hole in the bottom of a vessel. If the fluid at the beginning is entirely at rest it runs out without any gyrations; but if there is the least perceptible initial gyratory motion, it runs into very rapid gyrations near the centre.

34. In the case of tornadoes, which are always of small extent, the influence of the earth's rotation in producing gyrations is generally very small in comparison with that of the initial state of the atmosphere, so that the gyration depends principally upon the initial gyratory state of the atmosphere with regard to the centre of the tornado, and may be either from right to left or the contrary. Hence there may be tornadoes at the equator, although there cannot be large cyclones. In large cyclones the effect of the initial state, except at the equator, is insignificant in comparison with the influence of the earth's rotation; and the latter, moreover, is a constant influence, while the former is soon destroyed by resistances. Hence large cyclones are of long duration, while small tornadoes, depending principally upon the initial gyratory state for their violence, are soon overcome by the resistances.

35. On account of the centrifugal force arising from the rapid gyrations near the centre of a tornado it must frequently be nearly a vacuum. Hence, when a tornado passes over a building, the

exterior pressure, is in a great measure, suddenly removed, when the atmosphere within, not being able to escape at once, exerts a pressure upon the interior of perhaps nearly fifteen pounds to the square inch, which causes the parts to be thrown in every direction to a great distance. For the same reason, also, the corks fly from empty bottles, and everything with air confined within, explodes.

36. When a tornado happens at sea, it generally produces a water-spout. This is generally first formed above, in the form of a cloud, shaped like a funnel or inverted cone. As there is less resistance to the motions in the upper strata than near the earth's surface, the rapid gyratory motion commences there first, when the upper strata of the agitated portion of the atmosphere have a tendency to assume somewhat the form of the strata in the case of no resistance. This draws down the strata of cold air above, which, coming in contact with the warm and moist atmosphere ascending in the middle of the tornado, condenses the vapor and forms the funnel-shaped cloud. As the gyratory motion becomes more violent, it gradually overcomes the resistances near the surface of the sea, and the vertex of the funnel-shaped cloud gradually descends lower, and the imperfect vacuum of the centre of the tornado reaches the sea, up which the water has a tendency to ascend to a certain height, and thence the rapidly-ascending spiral motion of the atmosphere carries the spray upward, until it joins the cloud above, when the water-spout is complete. The upper part of a water-spout is frequently formed in tornadoes on land.

When tornadoes happen on sandy plains, instead of water-spouts, they produce the moving pillars of sand which are often seen on sandy deserts.

37. The routes of cyclones in all parts of the world, which have been traced throughout their whole extent, have been found by Redfield* to be somewhat of the form of a parabola, as represented in figure 3. Commencing generally near the equator, the cyclone at first moves in a direction only a little north or south of west, according to the hemisphere, when its route is gradually recurvated towards the east, having its vertex in the latitude of the tropical calm-belt. The motion of a cyclone toward the poles may be accounted for by the principle in section 5. The motion of the equatorial side of a cyclone in either hemisphere is always toward the east, and hence the deflecting force causes a pressure toward the equator, but that of the polar side being always toward the west, the deflecting force causes a pressure toward the pole. Now these deflecting forces being as the sine of the latitude, as may be seen from section 5, the pressure on the polar side toward the pole is greater than on the other side toward the equator, and hence the cyclone moves in the direction of the greatest pressure. It is not to be supposed, however, that there is an actual transfer of all the atmosphere of a cyclone from the equator to the polar regions. For the motions and pressure of the cyclone being greater on the polar side, where the deflecting forces which cause it are greatest, its action upon the atmosphere in advance of it is greater than on the equatorial side, where these forces are much less, and hence new portions of the atmosphere are being continually brought into action on one side, while the resistance of the earth's surface, and the adjacent portions of atmosphere on the other side, are continually overcoming the comparatively weak forces there, and destroying the gyratory motion of the cyclone; so that the centre of the cyclone is being continually formed in advanced portions of the atmosphere. Since many cyclones are more than one thousand miles in diameter, the difference in the violence of its action on the two sides is very considerable. Hence the interior and most violent portion of a cyclone always gyrating from right to left in the northern hemisphere, and the contrary in the southern, must always gradually move towards the pole of the hemisphere in which it is. While it is between the equator and the tropical calm-belt, it is carried westward by the general westward motion of the atmosphere there, but after passing the tropical calm-belt, the general motion of the atmosphere carries it eastward, and hence the parabolic form of its route is the resultant of the general motions of the atmosphere, and of its gradual motion toward the pole.

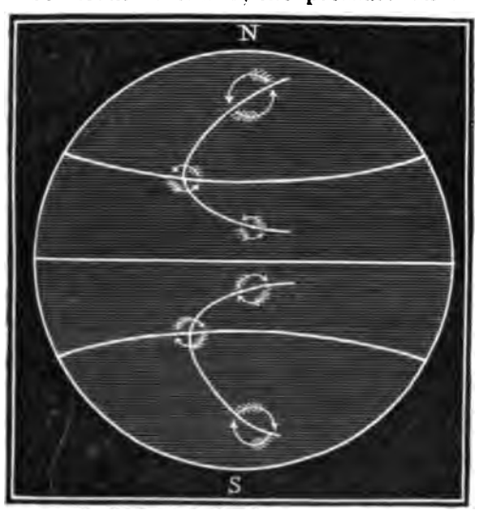

Figure 3.

* See this journal, [American Journal of Science,] second series, vol. i,. chart i.

IV. *The Motions of the Ocean.*

38. Besides the actions of the sun and moon which give rise to the tides, there are only two causes which can produce any sensible motions of the waters of the ocean. One of these is the action of the atmosphere upon the surface of the ocean, and the other, the difference of density between the water near the equator and that towards the poles, arising from a difference of temperature. The general motions of the atmosphere at the surface of the ocean have a tendency to cause a westward motion of the water in the torrid zone, and an eastward motion in the middle and higher latitudes, and from what we know of the effects of strong winds upon the ocean, we have reason to think that these general motions of the atmosphere are adequate to produce *sensible* motions, since, after the inertia of the water is overcome, which, however small the force, is only a question of time, the only force necessary is that which is adequate to overcome the resistance of friction, which is very small where the velocity is small. The difference of density between the equator and the poles causes a slight interchanging motion of the water between them, and consequently, where not interrupted by continents, it produces a system of motions in the ocean similar to those of the atmosphere. Hence these two causes of oceanic disturbance, whatever their relative weight, both act in the same directions and conjointly cause the observed westward motion of the ocean near the equator, and eastward motion toward the poles.

39. The westward motion of the water of the ocean in the torrid zone was first observed by Columbus, and is now well established, and observations also show that there is a motion towards the east in higher latitudes. A bottle thrown into the ocean near Cape Horn was picked up three and a half years afterward at Port Phillip, Australia, a distance of 9,000 miles, which makes the eastward velocity in that latitude more than seven miles per day. And Sir James Ross, when sailing near Prince Edward's Island, found himself every day from twelve to sixteen miles, by observation, in advance of his reckoning. ("Voyage to the Antarctic Seas," vol. ii, p. 96.) But a westward motion being established in the torrid zone, an eastward motion in the higher latitudes must be admitted; for, as was shown in the case of the atmosphere (section 13), the one cannot exist without the other.

40. It has generally been supposed that the equatorial westward current of the ocean is caused principally by the action of the westward winds there; but Professor Guyot thinks that "it is too deep and rapid to admit of being explained by their action alone," and that "the difference of temperature between the regions near the equator and those near the poles controls all other causes by its power and the constancy of its action." ("Earth and Man," pp., 189, 190.) The torsive or deflecting force which causes the westward motion of the atmosphere and the ocean in the equatorial regions, and the eastward motion in the higher latitudes, has been shown to be as the velocity of the interchanging motion between the equatorial and the polar regions; and hence if this motion in both were similar, the relative amount of this force in each must be as the whole mass multiplied into the velocity of this motion between the equator and the poles. If we suppose the ocean to be three miles in depth, its mass is about 500 times that of the atmosphere, and hence if the motion between the equator and the poles were only $\frac{1}{500}$ of that of the atmosphere, the part of the force which gives it a westward motion near the equator, and an eastward motion towards the poles, arising from this cause, must be greater than that of the action of the atmosphere upon it, since the whole amount of this force in the atmosphere is not spent upon the ocean, but only that part which overcomes the resistances to its motions. Although the effect of temperature in producing a difference of density, and consequently of disturbing the equilibrium, is very much less in the ocean than in the atmosphere, yet since the amount of motion which a given disturbing force will produce where time is not considered, depends as has been stated, upon the amount of the resistances, and not upon the amount of the inertia to be overcome; and since the resistances diminish as the square of the velocity, a very small amount of disturbing force arising from a difference of density must be adequate to cause an interchanging motion in the ocean between the equatorial and the polar regions equal to $\frac{1}{500}$ of that of the atmosphere; and hence we have reason to think that a greater part of the motions of the ocean is due to this cause than to the action of the atmosphere upon it.

41. The motions of the ocean being similar to those of the atmosphere, they must cause a slight elevation of the surface about the parallels of 30°, and a depression at the equator and the poles, just as in the case of the atmosphere, except that it will be less in the ratio of the relative velocities of the motions of the ocean and of the atmosphere. If we suppose the east and west motions of the ocean to be $\frac{1}{60}$ of those of the atmosphere at the earth's surface, which would require the maximum

eastward velocity in the southern hemisphere to be about ten miles per day, it would cause the surface of the ocean in the southern hemisphere to be about fifteen feet higher at the parallel of 30° than at the pole, and also a little higher than at the equator. Now if the motions which cause this accumulation of water were the same at the bottom of the ocean as at the surface, there would be no tendency of the water to flow out at the bottom from beneath this accumulation; but since the motions there must be much less, it must flow out both toward the equator and the pole, especially toward the latter, as the depression there is much the greater. Since the density of sea-water does not increase below the temperature of 28°, the density of the ocean does not increase beyond a certain latitude, and hence there is no flow of the water at the bottom from the poles toward the equator, arising from the maximum density at the pole, as seems to be the case in a very slight degree in the atmosphere, but the undercurrent at the bottom, arising from the greater pressure about the parallel of 30°, must extend entirely to the poles; so that there must be a slight tendency of the water to rise at the poles, and flow at the surface some distance toward the middle latitudes. As the water toward the bottom of the ocean is always about the same as the mean temperature of the earth, when it first rises to the surface at the pole, it must be much warmer than it is after it has flowed some distance from it, and hence we have reason to think that there may be open polar seas, surrounded by barriers of ice at some distance from the pole, where there is the maximum temperature of the surface water. A surface current from the poles is indicated by the motions of icebergs in both hemispheres from the polar regions toward a lower latitude.

42. Where the east and west motions of the ocean are entirely intercepted by continents, as in the northern hemisphere, the water receives a slight gyratory motion from the left to right. The westward motion of the waters of the Atlantic in the torrid zone, impinging against the continent of America, causes the surface of the water of the Caribbean sea and the Gulf of Mexico to be a little above the general level, while the eastward motion of the northern part of the Atlantic causes the surface of the water adjacent to the eastern coast of North America, in that latitude, to be a little lower. Hence, there is a flow of warm water from the Gulf of Mexico along the coast of the United States towards the lower level about Newfoundland, which, on account of the peculiar configuration of the coast about the Gulf of Mexico and the peninsula of Florida, gives rise to the Gulf-stream. The eastward motion, also, of the northern part of the Atlantic causes the surface of the water on the western coast of Europe to be a little *higher* than the general level, while the westward motion in the torrid zone causes it to be depressed on the western coast of Africa a little *below* this level, and hence the water of the eastern side of the Atlantic, flowing from a higher to a lower level, has a motion toward the equator. The whole of the north Atlantic has, therefore, a very slight gyratory motion from left to right, and is supposed to make a complete gyration in about three years.

43. A portion of the equatorial current flowing from the higher level of the Caribbean sea toward Cape Horn, causes the Brazil current, which is deflected eastward by the general eastward motion of the southern ocean. The east side of the south Atlantic, as well as that of the north Atlantic, seems to have a motion toward the equator. Says Sir James Ross, "There is a current from the Cape of Good Hope along the west coast of Africa sixty miles wide, two hundred fathoms deep, with a velocity of one mile per hour, of the mean temperature of the ocean."—("Voyage to the Southern Seas," vol. ii., p. 35). This cannot be a portion of the Mozambique current from the warm waters of the Indian ocean, passing around the Cape of Good Hope, and giving rise to the equatorial current of the Atlantic, as has been supposed, but must come from the colder waters of the southern ocean. Hence the south Atlantic also has a tendency to assume a gyratory motion, and the equatorial current of the Atlantic is merely the equatorial portion of these two gyrations, with perhaps a small part of the Mozambique current passing around the Cape.

44. The general eastward motion of the water of the northern part of the Atlantic, and the consequent depression of the water next the coast of North America, is the cause of the cold current of water flowing from Baffin's bay and the east coast of Greenland, between the Gulf-stream and the coast of the United States, called the Greenland current. Since the warm water of the Gulf-stream, in flowing northward, is deflected toward the east (section 5), and that of the Greenland current, in flowing south, tends toward the west, there is no intermingling of the waters of the two currents, but they are kept entirely separate as if divided by a wall, as has been established by the Coast Survey.

45. There must be a motion of the water somewhat similar to the Gulf-stream and the Greenland current, wherever the great equatorial current impinges against a continent, and the eastward motion

toward the poles is interrupted. Hence, on the eastern coast of South America there is the warm Brazil current which has been mentioned, and on the eastern coast of Asia there is the warm China current flowing toward the north, similar to the Gulf-stream, and the cold Asiatic current insinuating itself between it and the coast, like the Greenland current. On the east coast of Africa, also, there is the Mozambique current flowing south like the Brazil current, and it is also now well established that, east of the Cape of Good Hope, the general tendency of the water is toward the south. This water must mingle with the general eastward current of the south sea, and hence there is a slight tendency to a gyratory motion in the Indian ocean also.

46. On the western sides of the continents there is a motion somewhat the reverse of this, and instead of a warm current flowing north, there is a cold one flowing toward the equator, as has been shown to be the case in the Atlantic. Hence, on the west coast of North America there is a flow of colder water along the coast from the north, and on the west coast of South America is Humboldt's current, much colder than the rest of the ocean in the same latitude, both tending toward the equator to join the great westward current there across the Pacific, and to fill up, as it were, the vacuum which this current has a tendency to leave about the equator, on the west coast of America.

47. With regard to the gyratory motion of the oceans, it may be further added here that such gyrations are clearly demonstrated by the positions of the isothermal lines, as has been shown by Professor Dana, in a paper read at the twelfth meeting of the American Association for the Advancement of Science (Proceedings, vol. xii, p. 77). According to this paper the isothermal line of 68° Fahr., in winter, extends, in the north Atlantic, from 56° N. on the American side to 12° N. on the African; and in the south Atlantic, from latitude 31° S. on the South American coast to 7° S. on the African side. Similar evidences are given of gyratory motions, in a less degree, in both the north and south Pacific, and also in the Indian ocean.

48. When a portion of fluid on the earth's surface gyrates from left to right, the deflecting force arising from the earth's rotation being in this case toward the interior, the surface assumes a slightly convex form. The water of the north Atlantic having a very small gyratory velocity in comparison with that of the earth's rotation, the interior is a little elevated above the general level, and consequently the pressure upon the bottom increased. Now the gyrations which cause this elevation in the middle being principally toward the top, the increased pressure upon the bottom causes the fluid there to flow out on all sides with a very small velocity, towards the circumference, and hence the water at the surface has a slight tendency to flow in from all sides towards the interior to supply its place. This completely accounts for that vast accumulation of drift and sea-weed, covering a large portion of the interior of the north Atlantic, called the Sargasso sea. From what has been stated, the north Pacific must also have a slight gyratory motion from left to right, and hence it likewise has its Sargasso sea.

V. *The Motions of Solid Bodies at the Earth's Surface.*

49. It has been shown that if a body were set in motion upon the earth's surface it would move with uniform velocity, but would be continually deflected to one side. When the range of motion is small, $\cos\theta$ (equations (5)) may be regarded as constant, and hence the deflecting force in this case is constant, and must cause the body set in motion to describe the circumference of a circle. If we put ρ for the radius of curvature, and m for the angular velocity about the centre of curvature, the centrifugal force of the body is $\rho\, m^2$ which must be put equal to the deflecting force, (equation 5.)

Hence we have

$$\rho\, m^2 = 2\, n\, v \cos\theta.$$

Also, since v is the lineal velocity of the moving body,

$$\rho\, m = v.$$

From these two equations we get

$$\rho = \frac{v}{2\, n \cos\theta},$$

$$m = 2\, n \cos\theta.$$

50. When the range of motion is so small that $\cos\theta$ may be regarded as constant, ρ and m are constant, and hence the body then moves with a uniform angular velocity in the circumference of a circle. If we put T for the time of a revolution, we shall have

$$T = \frac{2\,\pi}{m} = \frac{\pi}{n \cos\theta} = \tfrac{1}{2} \text{ day} \times \sec\theta.^*$$

Hence since v disappears in the result, T is independent of the initial velocity.

* This result was erroneously given in the "Mathematical Monthly," 1 day × sec. θ.

51. If a body is forced to move in a straight line, F (equation (6)) is the lateral pressure of that body. If we put $V=60$, which is a velocity of about 40 miles per hour, the equation gives $F=\frac{1}{5188}g$, at the parallel of 45°. Hence if a railroad train moves in a straight line 40 miles per hour at the parallel of 45°, the lateral pressure is $\frac{1}{5188}$ of its weight, and this is precisely the same in all directions, and not in the direction of the meridian only as has been generally supposed.

52. The deflecting force (section 5) also causes the gyration of a vibrating pendulum. If the pendulum were suspended at the pole it would evidently vibrate in the same plane in space, and consequently, perform a gyration in one day. Since the velocity of the earth's revolution around any other point of the earth's surface is $n\cos\theta$ (section 28), the time of gyration there is $\frac{1\text{ day}}{\cos\theta}=1\text{ day}\times\sec.\theta$.

53. This same deflecting force may be used to explain some of the motions of a rotating body. Suppose such a body be placed with its axis of rotation in any direction parallel with the horizon. Then the motions of the upper and lower parts being in contrary directions relative to the earth and the deflecting force in both being either to the right or the left, according to the hemisphere in which it is, gives the axis of rotation a tendency to assume a perpendicular position. But there are other forces besides these horizontally deflecting forces, so that all the forces which tend to change the position of the axis would not be in equilibrium with the axis in that position. For the equatorial side then would have a motion coinciding in direction with the motion of the earth's rotation, while the other side would have a motion the contrary way, and, consequently, the centrifugal force arising from the motion of the earth's rotation, combined with that of the rotating body would be greater on the equatorial than on the polar side, and give the axis of the rotating body a tendency to move in the plane of meridian. It might be easily shown, when the axis has a position parallel with the axis of the earth, that the forces which tend to change its direction are then in equilibrium, and consequently the axis, if free to turn in any direction, does not change its position.

54. These deductions from theory are in exact accordance with some very delicate experiments made by Foucalt with a peculiar form of gyroscope, an account of which is given in this journal, second series, vol. xv, page 263. See also vol. xix, page 141.

No. 3.

THE CAUSE OF LOW BAROMETER IN THE POLAR REGIONS AND IN THE CENTRAL PART OF CYCLONES.

BY PROFESSOR WILLIAM FERREL.

[Extracted from "Nature," Vol. iv., page 226.]

In none of the treatises on meteorology or physical geography is there to be found any satisfactory explanation of the observed low barometer in the polar regions, or in the center of a cyclone. Observations show that in the antarctic region there is a permanent depression of more than one inch below the average height nearer the equator, and in the arctic region a depression of about half that amount; and also that for several days frequently the barometric pressure of the central part of a cyclone is one or two inches less than that of the exterior part. Mr. Buchan, in his excellent treatise on meteorology, attributes the low barometer in the polar regions to the effect of the vapor in the atmosphere. If the amount of vapor in the polar regions was greater than in the equatorial, this effect, so far as it would go, would be in the right direction; but just the reverse is the case; for it is well known that the amount of vapor in the warm equatorial regions is much in excess of that in the cold polar regions. Attempts have also been made, without success, to account for the depression in cyclones, by the effect of centrifugal force.

By whatever cause so great a difference in the barometric pressure in the different regions might be produced, it may be shown from the principles of dynamics that the equilibrium would be restored in a very short time, if there was not some constant force tending to drive the atmosphere from the polar regions to the equator, or from the center of the cyclone to the exterior, and to keep it in that position. Such a force may be found in the influence of the earth's rotation. In a paper by the writer, in the "Mathematical Monthly," in 1860, published in Cambridge, Mass., a full abstract of which was published in the January number of "Silliman's Journal" for 1861, the following very important principle was demonstrated: In whatever direction a body moves on the surface of the earth, there is a force arising from the influence of the earth's rotation, which tends to deflect the body to the right in the northern hemisphere, and to the left in the southern hemisphere. This force, which is the key to the explanation of many phenomena in connection with the winds and currents of the ocean, does not seem to be understood by meteorologists and writers on physical geography. We see it frequently stated that the drift of rivers and currents of the ocean running north or south always tends to the right in our hemisphere, and that a railroad car running north or south presses to the right; and this the case. But the same is true, and to exactly the same amount, of a current or of a railroad car running east or west, or in any other direction.

The amount of the deflecting force, when the velocity of the body is small in comparison with that of the earth's rotation, is expressed by $2.\frac{1}{289}.\frac{v}{n}\cos\theta\, g$; in which v is the lineal velocity of the body relatively to the earth's surface, n that of the earth's rotation at the equator, θ the angle of the polar distance, and g the force of gravity. If the velocity is expressed in miles per hour, the expression in round numbers becomes $\frac{v\cos\theta}{150{,}000}g$; that is, for each mile of velocity per hour, the force is $\frac{1}{150000}$ of gravity, multiplied into the cosine of the polar distance. Hence a railroad car on the parallel of 45° north, running in any direction at the rate of forty miles per hour, presses to the right with a force equal to about $\frac{1}{5000}$ part of its weight.

The effect of this deflecting force upon what Mr. Stevenson calls the barometric gradient is easily estimated. Since the strata of equal pressure of the atmosphere, so far as this force is concerned,

must be perpendicular to the resultant of this force and gravity, the sine of inclination of any such stratum to the earth's surface must be $\frac{v \cos \theta}{150,000}$, and the change in barometric pressure for any given distance is equal to the weight of a column of atmosphere of a height equal to the change of level of the stratum of equal pressure, and of a density equal to that at the earth's surface. The barometric gradient, then, as expressed by Mr. Stevenson, for any distance d expressed in miles is $\frac{r \cos \theta \, d}{51 \times 50,000} \times 30$ inches; putting five miles for the height of a homogeneous atmosphere, and thirty inches for the pressure at the earth's surface. Round numbers are used throughout, since it is only the order of the effects we wish to determine, and not their exact amounts.

According to all observations, there is a steady and very strong wind blowing all around the earth in the middle and higher latitudes of the southern hemisphere, with a velocity of at least twenty-five or thirty miles per hour at the surface of the ocean, and this is perhaps much greater in the upper strata of atmosphere. If at the parallel of 50° we suppose the velocity of the wind v to be thirty miles per hour, the preceding expression of the barometer gradient for a distance d, of 5° or three hundred and fifty miles, using the cosine of 40°, is 0.33 inches of mercury. By reference to section 113 of Mr. Buchan's Meteorology, it will be seen that the barometric gradient for that parallel is only 0.28 for 5° of latitude, and that this is about the maximum gradient in the southern hemisphere. Hence a velocity less than thirty miles per hour at the surface of the sea, especially if we suppose that it increases in the higher regions, is sufficient to account for this maximum barometric gradient; and according to observations, twenty or thirty miles per hour for the wind in that region is no unreasonable assumption. The eastward velocity of the wind in the different latitudes being known, and consequently, the corresponding barometric gradients, the difference of barometric pressure between any parallel near the pole and one toward the equator, is readily obtained by integration. As the wind near the equator is toward the west, the deflecting force there is *toward* instead of *from* the pole, and hence the greatest barometric pressure is about the parallel of 30°, and there is a slight depression at the equator. The deflecting force and the consequent depression are small there, on account of the small value of cos θ near the equator.

Since there is more land and mountain ranges in the northern than in the southern hemisphere to obstruct the eastward motion of the atmosphere, its velocity is not so great, and consequently the polar depression is much less there than in the southern hemisphere. According to Mr. Buchan, the barometric depression in the arctic regions is much greater in the northern part of both the Atlantic and Pacific oceans, than it is in the same latitudes on the continents. The explanation of this is, that the eastward velocity of the atmosphere over the oceans being much greater than it is on the continents, where it is obstructed more by friction and mountain ranges, the force driving the atmosphere from the poles toward the equator is less, and consequently the barometric pressure is less in the northern part of both oceans than it is on the continents in the same latitudes.

Upon the relative strength of the forces tending to drive the atmosphere from the poles towards the equator, depend the positions of the equatorial and the tropical calm-belt. This force being strongest in the southern hemisphere on account of less resistance from friction and mountain ranges, the mean position of the equatorial calm-belt is a little north of the equator, and the positions of the others a little farther north than they would otherwise be. The prime motive power, also, in both hemispheres being the difference of density of the atmosphere between the polar and the equatorial regions, arising from a difference of temperature and of the amount of aqueous vapor, during our summer; when this difference is less than the average in the northern hemisphere and greater in the southern, these calm-belts are forced a little north of their mean positions; of course, just the reverse of this happens during our winter; hence we have an explanation of the annual variations of the positions of these belts.

In the case of cyclones, the atmosphere at the earth's surface being forced in from all sides toward the centre, by the force arising from a difference of density of the atmosphere in the central and exterior parts, it cannot, on account of the deflecting force which has been explained, move toward the centre, without, at the same time, receiving a gyratory motion around that centre. Neither can it have a gyratory motion without also having a motion toward that centre, since in that case there would be no force to overcome the frictions of gyration. Hence, neither the radial theory of Espy, nor the strictly gyrating theory of Reid and others can be true, though either of them may be approxi-

mately so in special cases. But the gyratory part of the motion is not caused by the motion of the atmosphere from the north and south only toward the centre of the cyclone, as stated by Mr. Buchan and others, but equally by the different parts moving in from all sides, since in whatever direction they move toward the centre there is the same deflecting force, either to the right or the left according to the hemisphere.

The motion of the atmosphere being in a spiral toward and around the centre of the cyclone, the deflecting force depending upon the earth's rotation, at right angles to the direction of motion, being resolved in the directions of the radius of gyration and tangent, the latter overcomes the friction of gyration, and the former causes a pressure from the centre, decreasing the height of the strata of equal pressure in the cyclone, and consequently diminishing the barometric pressure. The barometric gradient of a cyclone is estimated in precisely the same way as in the case of the hemispheres, using for v the lineal velocity of gyration obtained by resolving the real motion into the directions of the tangent of gyration and of the radius. It has been seen that a velocity of thirty miles per hour gives a barometric gradient of $\frac{1}{3}$ of an inch in three hundred and fifty miles on the parallel of 50°. A gyratory velocity therefore of one hundred miles per hour would give a barometric gradient of one inch of mercury in about three hundred miles. The velocities of gyration being known at all distances from the centre of motion, and consequently the barometric gradients, the difference of barometric pressure between the centre and the exterior, so far as it depends upon the gyratory motion, may be obtained by integration. The effect of the centrifugal force of the gyrations is generally only a very small quantity of a second order, in comparison with the others, and the effect of it is entirely insensible, except in the case of small tornadoes, when the gyrations are very rapid close around the centre.

In all the preceding estimates of the barometric gradient, it should be understood that the results belong merely to the force depending upon the earth's rotation, and to this must be added the part belonging to a difference of density of the atmosphere, which, in the case of cyclones, increases the gradient, but diminishes it in the case of the hemisphere. For the general motions of each hemisphere form a cyclone, with the pole as a centre, but having the denser instead of the rarer portion of the atmosphere at that centre. Hence the motions in any vertical plane through the centre are reversed, and it becomes what has been called an anti-cyclone.

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NO. 4.

RELATION BETWEEN THE BAROMETRIC GRADIENT AND THE VELOCITY OF WIND.

BY WM. FERREL.

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1. Let G = the barometric gradient in the direction in which it is the steepest, estimated by the amount of change in the mercurial column in the distance of 100 miles;
v = the velocity of the wind per hour;
r = the radius of curvature of the isobar or line of equal barometric pressure;
i = the inclination of the direction of the wind to the isobar on the side of lowest pressure;
n = the earth's hourly angular velocity of rotation in terms of the radius;
l = the latitude of the place;
P = the barometric pressure of the atmosphere;
P′ = the value of P at the earth's surface.

The following equation then expresses very nearly the relation in all cases between the barometric gradient and the velocity of the wind:

$$G = \frac{(2\,n \sin l + u)\, v \sec i}{8300000} \cdot \frac{P}{P'} = \frac{(0.524 \sin l + u)\, v \sec i}{8300000} \cdot \frac{P}{P'}, \tag{1}$$

in which

$$u = \frac{v \cos i}{r}. \tag{2}$$

This relation expresses a general law which is of as much importance in meteorology, so far as the barometric pressures and the velocities of the winds are concerned, as Kepler's laws were in astronomy, and must hold for all latitudes from the equator to the poles both at sea and on land, and for all altitudes from the earth's surface even beyond the region of the cirrus clouds. It may be applied to the mean constant motions and pressures of the atmosphere depending upon the mean difference of temperature between the equator and the poles, unaffected by the seasons, which gives rise to two grand hemispherical cyclones, of which the poles are the centres, and of which the cyclonic motions consist of the approximately eastward motions in the middle latitudes and the trade-winds in the torrid zones, and also to the ordinary cyclones of comparatively limited extent, including the violent tornadoes and water-spouts, all of which are cyclones contained within the larger cyclones and controlled by their motions. It is likewise applicable to the resultant of any number of cyclones contained within, or interfering with, one another.

2. From (2) it is seen that u is the angular velocity of gyration around the centre of curvature of the isobar, which in a perfect cyclone becomes the gyratory velocity around the centre of the cyclone. But in connection with this gyratory motion there is generally a motion either toward or from the centre of the cyclone, toward the centre below and from the centre above in an ordinary cyclone, giving rise to a spiral motion, and hence at a certain height the motion toward or from the centre must vanish and the gyrations be circular. The value of i, therefore, has different signs below and above.

The value of i depends mostly upon friction, but in some measure also upon the inertia of the atmosphere in cases where the motions are either increasing or decreasing, as in the beginning or dying away of the cyclone. Its value, therefore, is greatest on land and near the earth's surface, and comparatively small at sea or anywhere in the upper regions of the air. Its value also depends upon the sine of the latitude, and is equal to 90° at the equator, where $\sin l = 0$, since there can be no gyrations there, and the motions must be either toward or from a centre of rarefaction or condensation.

In a perfect cyclone the isobars are circular and r becomes the radius of the circle, and in this case the gradients are estimated in the direction of the radius from the centre. In the two polar

hemispherical cyclones the gradients are estimated in the direction of the meridians, and the isobars, supposing the cyclones to be perfect and unaffected by local disturbing causes, correspond with the parallels of latitude. In this case r in (2) is the distance from the earth's axis, and $v \cos i$ expresses the component of motion relative to the earth's surface in the direction of the parallels of latitude. In order to have the expression of G (1) strictly applicable to the polar cyclones we should have $u \sin l$ instead of u, but the value of u in comparison with $2n \sin l$, in this case, is so small that u may be neglected entirely without sensible error, so that the expression may be regarded as applicable either to the polar cyclones, or to any ordinary cyclone, or to the resultant of any number of these cyclones interfering with one another.

3. In ordinary cyclones or tornadoes, in which circumstances may be such as to give a very great gyratory motion near the centre, the term in (1) depending upon u cannot in general be neglected, and it may even become much greater than the term depending upon n, that is, upon the earth's rotation, and in violent tornadoes and water-spouts, in which there are rapid gyrations very near the centre, the value of the term depending upon u may be so much greater than that depending upon n, that the latter may be entirely neglected in comparison with the former. For instance, if the distance r from the centre of the cyclone should be 400 miles and the velocity v equal to 40 miles per hour, if we regard the value of i as being not very large, (2) would give $u = 0.1$, which in the middle latitudes would be about one-fourth of $0.524 \sin l$, and its omission would give rise to considerable error. And if the the distance from the centre were only 100 miles and the velocity of the wind the same, the two terms in the expression of G would be about equal, and for very small distances from the centre with a large value of v, it is readily seen that the term containing n and depending upon the earth's rotation may be very small in comparison with the other. In large cyclones, however, and at a considerable distance from the centre, the value G depends mostly upon the effect of the earth's rotation, and only in some measure upon the mere centrifugal force arising from the gyrations relative to the earth's surface around the centre, and depending upon the term containing u merely and not n.

4. The preceding general law expressed by (1) is deduced from carrying out more in detail principles which the writer has already had published at different times and places, but a complete demonstration of the law would be too complex and require too much mathematical analysis to be given here. It may, however, be important to give the following explanation, rather than a complete demonstration, of this law so far as it applies to ordinary cyclones of not very great extent, especially as the method of presenting the matter here is different from any which has heretofore been used.

If water in a basin at rest has a motion of gyration around the centre, the mere centrifugal force arising from the gyrations causes the water to recede from the centre and produce a gradient in the surface of the water, and consequently in the pressure of the fluid upon the bottom of the vessel. If we let r represent the distance from the centre and u the angular velocity of gyration, the centrifugal force is expressed by ru^2 simply. But if in addition to the gyratory motion of the water in the basin, the basin itself also has a gyration like a disk around its centre, of which the angular velocity is represented by n', we then have for the whole centrifugal force

$$r(n' + u)^2 = r(n'^2 + 2n'u + u^2),$$

and the gradient referred to the level of the water at rest in the basin, also at rest, depends upon this force. But if we refer the gradient to the level of the water at rest relatively to the basin having the gyratory velocity n', we must neglect the first term in the second member above depending upon n' merely, and the gradient then depends merely upon

$$r(2n'u + u^2) = (2n' + u)r,$$

putting $v = ru$ for the lineal velocity of motion or gyration in this case.

5. In the case of a cyclone upon the earth's surface of such extent only that the curvature of the surface can be neglected, we have exactly a similar case. In addition to the gyratory motion of the cyclone relative to the earth's surface, is is well known that the part of the earth's surface occupied by the cyclone has a motion, one component of which is that of the gyration of a disk around its centre. At the poles of the earth, the angular velocity of this gyration is that of the earth's rotation around its axis represented by n, but for any place between the poles and the equator it is equal to the velocity of the earth's rotation multiplied into the sine of the latitude, that is, equal to $n \sin l$. In this case we have $n \sin l$ corresponding to n' in the case of the basin of water, and hence we have $(2n \sin l + u)r$ for the centrifugal force upon which the gradient of the cyclone depends, for the term in this case depending upon $n^2 \sin^2 l$, corresponding to n^2 in the case of the basin of water, must be

neglected, as in that case, since the atmospheric gradient is referred to the elliptic surface of the earth, and not to the surface in the case of no rotation around its axis. This corresponds with the centrifugal force in (1), upon which the gradient depends in the case of no friction, in which the gyrations are circular, as supposed in the basin of water, and in which consequently sec $i=1$.

6. If we put

h = the height of any stratum of the atmosphere of equal pressure;

ρ = the density of this stratum;

ρ' = the value of ρ at the earth's surface;

we shall have

$$\frac{D_r P}{\rho} = g\, D_r h, \tag{4}$$

and, putting a for 100 miles, we get, according to the definition of G,

$$G = \frac{a\, D_r h}{10500} \cdot \frac{\rho}{\rho'}, \tag{5}$$

in which 10500 is the ratio very nearly between the density of the atmosphere and of mercury at the surface of the earth, where the atmospheric pressure is supposed to be 30 inches and the temperature equal to 32° of Fahrenheit. When the mean temperature of the atmosphere is greater, the value of this constant should be increased $\frac{1}{490}$ for each degree of temperature.

With the value of $D_r h$ obtained from (5) we get from (4)

$$\frac{D_r P}{\rho} = 10500 \frac{g}{a} G. \frac{\rho'}{\rho}.$$

But the first member of this equation is the expression of the horizontal accelerating force arising from a difference of pressure, and this in the case of no motion either toward or from the centre of the cyclone, as in the case of the basin of water, must be equal to the part of the centrifugal force causing the gradient, which, when the gradient is referred to the elliptic surface of the earth, we have seen, is $(2n \sin l + u)v$. Hence we get in this case

$$\frac{D_r P}{\rho} = 10500 \frac{g}{a} G. \frac{\rho'}{\rho} = (2\, n \sin l + u)\, v.$$

Where there is motion toward or from the centre of the cyclone we must add a term, F, to the last member of this equation, to represent the frictional resistance of the motion, and likewise one to represent the inertia of the air where its motion is either accelerated or retarded. On account of the extreme mobility of the air this last term may be generally neglected without any sensible error in any of the usual motions of the atmosphere, for it can be shown that only a very small part of the observed barometric gradients are necessary to overcome the inertia in accelerating and retarding the observed velocities. Neglecting, therefore, this effect, and adding F to the last member of the preceding equation, we get

$$G = \frac{g}{a} \cdot \frac{(2n \sin l + u)\, r \cos i + F}{10500} \cdot \frac{\rho}{\rho'}. \tag{6}$$

Since the motion is now spiral and not circular, and the centrifugal force depends simply upon the component belonging to circular motion, we must use here $v \cos i$ instead of v simply in the preceding expression.

7. By the principle of the preservation of areas in the case of central forces only and no friction, we would have in all parts of the cyclone

$$r^2 (n \sin l + u) = \text{constant}.$$

Hence we get by differentiation

$$-r\, D_t u = 2 (n \sin l + u)\, D_t r.$$

The second member of this equation expresses the force which tends to produce a gyratory motion around the centre of the cyclone. In the case of no friction this force is all spent in accelerating or retarding the gyratory velocity as the particles of air approach or recede from the centre, but where there is friction, it is mostly spent in overcoming the frictional resistance. We shall, therefore, have very nearly

$$F' = -2 (n \sin l + u)\, D_t r. \tag{7}$$

putting F′ for the resistance to motion at right angles to the radius, or in the direction of the gyratory motion.

The resistance to the horizontal motion of any stratum of atmosphere which has to be overcome by the existing forces consists in the difference of the actions through friction of the stratum immediately above and below that stratum, so that when the relative velocity of either two contiguous

strata is the same there is no resistance to be overcome by the forces. In the motion of the winds the velocity of the lower stratum relative to the earth's surface is greatest, the velocity of the second relative to the first a little less, and so on, until at a moderate height above the earth's surface the relative velocities, and consequently the resistance of friction, are very small. The frictional resistance, therefore, which has to be considered, belonging to each stratum or particle, is principally near the earth's surface, and at a small distance above it becomes comparatively small. Near the earth's surface, where the velocities of the strata increase with the height, this resistance is in the contrary direction of the motion of the atmosphere, and the component of this resistance, contrary to the direction of the gyratory motion, of which the velocity is $ru \cos i$, is represented by F′. The other component of the resistance, therefore, contrary to the direction of the radius, in which the velocity of motion is $D_t r$, is represented by

$$F = F' \cdot \frac{D_t r}{ru \cos i} = \frac{2 (n \sin l + u) (D_t r)^2}{ru \cos i}.$$

This expression is always positive, but it applies only to the part near the earth's surface, where the one component of motion is toward the centre of the cyclone.

With this value of F (6) gives, neglecting $\frac{1}{2} u$ in comparison with $n \sin l$ in the value of F′,

$$(8) \qquad G = \frac{a}{g} \cdot \frac{(2 n \sin l + u)}{10500} v \cos i \left(1 + \frac{(D_t r)^2}{(v \cos i)^2}\right) \cdot \frac{\rho}{\rho'},$$

$$= \frac{a}{g} \cdot \frac{(2 n \sin l + u)}{10500} v \sec i \cdot \frac{\rho}{\rho'},$$

since by the definition of i we have

$$(9) \qquad \tan i = -\frac{D_t r}{v \cos i}.$$

Since the unit of time is one hour in the expression of G, we must put

$$g = 3600^2 \times 32.2 \text{ ft.} = 79040 \text{ miles.}$$

With this value of g, putting $a = 100$ miles, and with the value of n, the angular rotatory velocity per hour of the earth's rotation in terms of the radius, which is 0.262, we get from (8) by putting $\frac{\rho}{\rho'} = \frac{P}{P'}$, the expression of G in (1). At the earth's surface the factor $\frac{P}{P'}$ becomes unity.

8. If the internal and external parts of the cyclone had the same temperature, the strata of equal pressures would be parallel, or equidistant, and $D_t h$ would be a constant for the same place at all altitudes, and G would be proportional to S. But in order to keep up the motive power of the cyclone there must be a difference of temperature between the internal and external parts, and this causes an increase or decrease in the value of $D_t h$ with the altitude, and consequently by (5) an increase or decrease of G in a greater ratio than that of ρ, other things remaining the same. In an ordinary cyclone, in which the temperature is greatest, and consequently the density least, in the interior of the cyclone, the value of G increases in a less ratio than ρ, and hence in order to satisfy (8) $v \cos i$ must have a less value above than below; but the reverse of this is true where the density is greatest in the interior at the same pressure, as in the polar hemispherical cyclones, and hence the mean constant motions of the upper strata of the atmosphere relative to the lower ones is eastward in all latitudes.

9. The value of i in (8) depends upon friction, and its value can only be determined from observation. If in (7) F′ vanishes, $D_t r$ must vanish, and consequently by (9) i vanishes and the gyrations are circular. The greater the value of F′ for the same velocity or value of r, the greater must be the value of $D_t r$, and consequently of i. If for different velocities friction increases as the velocity, $D_t r$ must increase as the velocity increases, in order to satisfy (7), and hence (9) in this case gives i a constant for all velocities, so long as u can be neglected in comparison with $n \sin l$ in (7), but near the centre of the cyclone, where u becomes very large in comparison with $n \sin l$, $D_t r$ must become very small, other things remaining the same, and hence i must also become small. For all ordinary winds it is probable that the value of i is nearly constant for all velocities, and if so, being once determined from observation for a given kind of surface, as that of the sea, or for any kind of land surface, as that of a prairie, this value is probably applicable to cyclones of all degrees of violence upon such surface, except near the centre, and may be regarded as a known and fixed constant for that kind of surface.

10. Some very important work has been done by Rev. Clement Ley to determine the value of this constant, the results of which have been given in the Journal of the Scottish Meteorological Society for the first quarter of 1873, page 66. The following mean values of i, given in connection with the several stations in the following table, were deduced from a considerable number of observations taken indis-

criminately by comparing the directions of the wind with that of the isobars, as given by observations made in the several countries to which the stations belong:—

	° ′		° ′		° ′
Scarborough	4 58	Thurso	15 4	Nottingham	27 44
Brest	7 25	Holyhead	18 4	Oxford	29 12
Scilly	10 1	Aberdeen	21 3	Brussels	29 57
Yarmouth	13 49	London	21 7	Paris	36 23
Pembroke	14 47	Greencastle	22 1	Skudnesnaes	41 17

From these results Mr. Ley arrived at the following conclusions:—

I. The winds commonly incline from the districts of higher toward those of lower pressure. The collective mean for the 15 stations is 20° 51′.

II. This inclination is much greater at inland than at well-exposed coast stations. The collective mean for Brest, Scilly, Yarmouth, Pembroke, and Holyhead is 12° 49′, while for the internal stations, London, Nottingham, Oxford, Brussels, and Paris, it is 28° 53′.

These results exactly confirm the theory of ordinary cyclones, which requires, where there is friction, that there should be a motion of the air below toward the centre of the cyclone, as well as a motion of gyration, and hence i must have a positive value. This, it is seen, is obtained from observation for each one of the fifteen stations taken separately. Moreover, the inland stations, where the resistances are greater, give a greater value of i than the stations on the sea-coast, where the resistances are smaller. This likewise accords with theory.

From the small value of i obtained by Mr. Ley for the coast stations, we may infer that at sea, and likewise in the upper regions of the atmosphere, it is still considerably less. The value of i, therefore, in (8), except at internal stations where the resistances are great, may be regarded so small that its secant can be taken as unity, and hence either the gradient or the velocity of the wind is known the one from the other. For inland stations near the surface, where the resistances are great, the value of i must be obtained from observation.

11. With regard to the inclinations of the winds to the isobars belonging to the different quarters of the compass, Mr. Ley obtained for the inclination of se. winds 35° 11′, of ne. winds 17° 43′, of nw. winds 9° 4′, of sw. winds 20° 13′. In these results s. winds were taken as sw. winds, e. winds as se. winds, &c. Hence ese. winds have the greatest, and wnw. winds the least inclinations to the isobars, which correspond very nearly to the ne. and sw. sides of the cyclones. Mr. Ley states that the average direction of the cyclones was about ne. Hence it appears that the inclination of the front part of the cyclone is greatest, and that of the rear the least, and this may perhaps be found to be a general law. It was also found that the differences in the inclinations of the winds from the different quarters is greatest at coast stations.

Mr. Ley likewise obtained the very important result that the difference between the inclination of strong and light winds at stations on the coast is trivial, but at inland stations the mean inclination is less with strong than with light winds, and that at all stations the inclination is more regular with a gale than with a light wind. This is all exactly in accordance with the preceding theory, for we have just seen in a preceding paragraph that where the value of u in (7) becomes large, as it does near the centre of a cyclone, the value of i must become less; but the strongest winds are also near the centre of the cyclone, and hence, in general, the strongest winds are found to have the least inclination to the isobars. And as the tendency of the very rapid gyrations near the centre is to approximate to a circular gyration, it is evident that the inclinations must be more regular with such winds, which are generally gales, than with light winds; as was found to be the case by Mr. Ley from observation.

12. If the water in a basin of water has an interchanging motion between the internal and external parts, and this basin itself has a horizontal gyratory motion around its centre, it is well known that the water toward the centre, by the principle of the preservation of areas, tends to run into rapid relative gyrations, but if the basin has no such gyratory motion, the water does not run into such gyrations, but simply keeps the gyrations belonging to the basin. So if, by means of the rarefaction of some area of the atmosphere, so as to cause a difference of density between the internal and external parts, an interchanging motion is kept up between these parts, the atmosphere must run into gyrations, that is, give rise to a cyclone, if the area occupied by this part of the atmosphere has a gyration around its centre, as we know it has, unless this centre is on the equator. The value of u, therefore, in (7) depends upon the amount of gyratory motion which the part of the surface of the earth occupied by the cyclone has around its centre, and vanishes at the equator where this gyratory

motion vanishes. The factor ($n \sin l + u$), therefore, in the second member of (7), depends upon the latitude of the place, and vanishes at the equator. Hence the greater the latitude the less must be the value of D, r, when $v \cos i$ and consequently F′, the resistance to this component of the motion, remain the same; and therefore by (9) the less must be the value of i. At the equator also, where $r \cos i$ vanishes on account of the vanishing of the gyrations, tan i becomes infinite, and consequently $i = 90°$.

13. Mr. Ley's results obtained from the averages of all the stations correspond to about the parallel of 50°. We now see that, according to theory, the less the latitude the greater must be the value of i obtained from observation, and that at the equator the motions would be found at right angles to the isobars. It would, therefore, be interesting to have the value of i obtained from observations for other latitudes. This has been done recently by Professor Loomis, (American Journal of Science, July, 1874), from two years' observations of the Signal Service of the United States, as given in one of the tri-daily weather maps for each day. The method adopted is different from that used by Mr. Ley, but the results obtained should be the same by each method. The angle of inclination obtained from these weather maps was over 45°, which is more than one-third of this value greater than that obtained by Mr. Ley for the five inland stations. The average latitude of the stations on the United States weather maps is considerably less than that of the five inland stations used by Mr. Ley, and a part of the preceding difference is, no doubt, due to that cause, as theory requires, but perhaps the greater part is due to the greater amount of frictional resistance to the gyrations in a new and wooded country.

In the case of the trade-winds, a part of the polar cyclones, the inclination of the direction of the wind to the isobars at sea is about 45°, and this being at about the latitude of 20°, the value of this angle, by theory, should be very much greater than its value at sea at the parallel of 50°, which from Mr. Ley's small value obtained for the coast stations is perhaps less than 10°, so that this also confirms the preceding theory.

14. Having now learned something from both observation and theory with regard to the value of the theoretically unknown angle i contained in the relation expressed by (1), we shall now proceed to make some comparisons of this law or relation with observation. We shall first consider the mean gradients and velocities belonging to the two polar hemispherical cyclones. Observation shows that the barometric pressure is a maximum, and that, consequently, G vanishes about the parallel of 35° in the northern hemisphere, and a little nearer the equator in the southern hemisphere, and that there are calm-belts, called the tropical calm-belts, at these latitudes, except so far as they are occasionally disturbed by local cyclones. By the relation expressed by (1), if we put $G = 0$, we must also have $r = 0$, unless cos i vanishes, which, we have seen, does not, except at the equator. There must, therefore, be a calm where $G = 0$, and hence we have the tropical calm-belts. At the equator we also have $G = 0$, and with this value (1) is satisfied with $r = 0$, that is, with a calm-belt, as observed at the equator; but it is likewise satisfied by ($2n \sin l + u$) vanishing at the equator, and hence r is arbitrary.

Again observation shows that the mean constant barometric pressure increases in the middle latitudes in going from the poles to the equator, and decreases within the tropics, and hence G is positive in the former case and negative in the latter. The relation of (1), or of (8), requires that in the former case we should have the principal component of velocity, $r \cos i$, positive, and in the latter case negative, that is, that in the middle latitudes there should be an eastward motion, and between the tropics and the equator a motion toward the west. This, it is well known, is in accordance with observation.

15. The barometric pressures given by Professor Loomis ("Meteorology," page 18) indicates that at about the parallel of 64° in the northern hemisphere, and about the parallel of 74° in the southern hemisphere, there is a minimum of barometric pressure, and hence G vanishes in these latitudes, and therefore, for reasons which have already been given, we must have calm-belts there. The observations from which these pressures have been deduced perhaps did not sufficiently embrace all longitudes and all seasons to give the mean constant pressures, unaffected by local circumstances and the seasons. This we know is the case in the southern hemisphere, where the most southern observations, obtained mostly by the British Board of Trade, were necessarily made during the summer season, when the barometric pressure is the greatest in those latitudes. If, however, it can be clearly shown by observation that the mean annual pressure is a minimum at these latitudes all around the globe, then the

gradient or value of G is negative between these parallels and the poles, and there must consequently be a wind between these parallels and the poles having the component $v \cos i$ from the east, and as the other component of motion at the surface must in this case be from the pole toward the minimum pressure, the wind must be from some point between the north or south, according to the hemisphere, and the east.

16. The real velocities and directions of the mean constant winds have been determined only very roughly from observation on any part of the globe, and hence no very accurate comparisons of our law with observation can be made. Such comparisons, however, all seem to establish the truth of the law within the limits of the errors of observation. The mean constant isobars in the British Islands, as determined by Mr. Glaisher, all effects of the seasons and of local disturbing causes being eliminated, gives very nearly $G = 0.02$ of an inch. With this value in (1), supposing the direction of the wind to be toward the east, or nearly so, or that the value of i is small, we get $v = 6$ miles, nearly. This is a very little less than the mean eastward velocity of the wind here, as determined by the late Professor Coffin, and given in his "Winds of the Northern Hemisphere." Again, from the table of barometric pressures given by Buchan ("Handy Book of Meteorology," page 27) we deduce the approximate value of $G = -0.02$ about the parallel of 20° in the northern hemisphere. The stations in this table belong to widely different longitudes, and the gradients, no doubt, differ considerably on the same parallel in different longitudes, so that this is merely a rough average value for that latitude. On the ocean in this latitude, where the trade-winds prevail, without any disturbance from local disturbing causes, the direction of the wind being about ne., is inclined to the isobars about 45°, and the direction being westward the value of i in this case is about $180° + 45° = 225°$. With this value of i, and the value of G above, we get from (1) $v = 15$ miles, nearly, the principal component $v \cos i$ being negative and consequently westward in this case. This velocity cannot differ very much from the usual velocity of the trade-winds, so that the result seems to confirm our law with regard to velocities.

From the table of barometric pressures given by Buchan, which has been already referred to, we obtain for the parallel of 52° in the southern hemisphere the value of $G = 0.07$ of an inch, and the tabular results from which this is obtained are the averages of a very large number of observations obtained by the British Board of Trade, so that this gradient may be regarded as being pretty accurate, and it is perhaps very nearly the same on that parallel all around the globe. Supposing the value of i here to be small, that is, that the wind blows very nearly from the west, we get from (1), with this value of G, $v = 21$ miles for the velocity of the winds on this parallel. All accounts represent the west winds in this latitude as being very strong all around the globe. Mr. Laughton says:* "Between the parallels of 40° and 60° the westerly wind blows with a constancy little inferior to that of the trade, but with much more violence. About the parallel of 50°, indeed, it is found as a rule, to be blowing 'half a gale of wind,' and this not only south of the Atlantic, but all round the world." A velocity of 21 miles per hour makes a pretty strong constant wind, and this theoretical velocity probably corresponds very nearly with the average wind in this latitude.

17. We come now to make some applications of the law to ordinary cyclones. Since in the centre of a cyclone the barometric pressure must be a minimum and consequently $G = 0$, in order to satisfy (1) in this case we must have $v = 0$, and hence there must be a calm at the centre. In the large cyclones the gradient or value of G may not become sensible for a considerable distance from the centre, and in such cases there is no sensible velocity of the wind within that distance. The area of almost a perfect calm in some cyclones is said to be as much as 30 miles in diameter.

The gyrations of the external part of a cyclone are necessarily in the contrary direction, and hence the component of gyratory motion $v \cos i$ must be negative, and consequently sec v in (1), and the sign of G becomes reversed. At some distance, therefore, from the centre of a perfect cyclone between the centre and the outer limit, the barometric pressure must be a maximum and G vanish. At this distance, therefore, by our law we have $v = 0$, that is, a calm. Hence areas of high barometer must generally be areas of calms.

18. If the isobars of a cyclone drawn to every tenth of an inch of the barometer reduced to sea-level are 100 miles apart, we have $G = 0.1$ of an inch. With this value of G, supposing the value of i to be so small that we can put sec $i = 1$, we get from (1) at the distance of 400 miles from the centre of the cyclone, or centre of curvature of the isobars, and on the parallel of 45°, $v = 29$ miles for the velocity of the wind; and this would be very nearly the actual velocity at sea, where the value

* "Physical Geography in its Relations to the Prevailing Winds and Currents," page 41.

of i is small. But if the value of i is 45°, which is nearly the value obtained by Professor Loomis from the average of all the stations of the United States Signal Service, then we get for the velocity of the wind, under the same circumstances, $r = 21$ miles. With a still much greater inclination than this average inclination, which must frequently happen, this velocity becomes much less.

At a distance of only 100 miles from the centre, all the other circumstances being the same as above, we get $v = 22$ miles in the case in which i is small, as at sea; and in the case in which the value of i is 45°, we get $v = 18$ miles, nearly. The comparison of these values of v with those of the preceding paragraph shows that the velocity of the wind belonging to the same gradient diminishes considerably toward the centre of the cyclone.

19. The preceding law or relation contained in (1) cannot be tested by comparing the observed velocities of the wind in individual cases with those deduced from the corresponding gradients obtained from the isobars as laid down on the weather maps of the Signal Service; for these being laid down from observations made at the stations which are in many cases several hundred miles apart, the effects of the minor more local disturbances cannot be taken fully into account, since within an ordinary cyclone there may exist several smaller cyclones with distinct centres of their own, of so small extent that their effects upon the isobars cannot be determined from stations merely at wide distances apart. The law can only be tested by comparing the average velocity of a great many individual cases with the average of the corresponding observed gradients, as deduced from the isobars, taking account of the distances of the stations from the centres of curvature of the isobars.

20. According to the empirical law of Dr. Buys Ballot, the wind blows at right angles to the line joining the highest and lowest pressures, or, in other words, in the direction of the isobars, and with a force proportional to the steepness of the gradient. This, so far as it represents the true law of nature, is wholly contained within the preceding theoretical and much more general law. Theoretically the direction of the wind can never exactly coincide with that of the isobars, but according to the results obtained by Mr. Ley the inclination to this direction may be small in the higher latitudes at sea, and on level prairie or mostly cultivated countries, where the winds are not obstructed by woodlands; but according to our theoretical law this angle, even at sea, must become large toward the equator, and accordingly we find that the trade-winds at the parallel of about 20° are inclined to the direction of the isobars about 45°. This law of direction then, as a general law applicable to all latitudes, is not even approximately true near the equator. With regard to the force or velocity of the wind, we see from a mere inspection of the expression of G in (1), that the velocity in all parts of the cyclone for the same latitude is not proportional to G, but is less near the centre of a cyclone; and this has likewise been shown (section 17) by obtaining numerical results in special assumed cases. It is, moreover, seen that in different latitudes the value of r corresponding to the same gradient, or value of G, must be nearly inversely as the sine of the latitude, especially at a considerable distance from the centre of a cyclone, where u in (1) is small in comparison with $2n \sin l$. For this reason the violence of the wind in a cyclone corresponding to the same gradient is much greater within the tropics than in the higher latitudes.

21. If we put

$G' = G$ divided by 100 miles;

$D =$ the barometric depression in inches at the earth's surface in the middle of a cyclone;

we shall have

$$D = \int_r G' = \int_r \frac{(0.524 \sin l + u)\, v \sec i}{830000000}, \tag{10}$$

in which r must be expressed in miles, and the integration be made from $r = 0$ to r equal its value when the barometric pressure is equal to some assumed value, as the mean, below which the depression is estimated.

It is well known that there is a depression of the barometer below the mean in the middle of all cyclones. In the polar regions, near the centres of the two polar hemispherical cyclones, the barometric pressure is considerably below the mean, near the north pole more than a half inch, and near the south pole about an inch, the depression at the latter being greater on account of there being but little land and few mountains in the southern hemisphere to obstruct the gyrations around the pole, upon which the depression depends. In the middle, also, of ordinary cyclones a depression below the mean of two or more inches is sometimes observed. Up to this time no meteorologist has accounted for these depressions. It has been attempted to account for them by means of the centrifugal force

arising from the gyrations relative to the earth's surface merely, neglecting the gyrations arising from the earth's rotation; but this in all cases gives an effect very much too small. For upon this principle we neglect the principal term in the expression of D (10) depending upon 0.524 sin l, and retain only that depending upon u, which we have seen, is generally very small in comparison with the former. In fact, in the two polar hemispherical cyclones u is so small in comparison with 0.524 sin l, that it can be entirely omitted without sensible error, so that by retaining merely this very small term we get from (10) no sensible depression of the barometer toward the poles at all. In ordinary cyclones, also, the effect depending upon u merely is very small in comparison with that depending upon the other term, except in violent tornadoes of small extent, in which the principal part of the gyrations is near the centre. By using the complete expression of D we obtain a complete and satisfactory explanation of all these depressions. When a cyclone is of large extent no great velocity and corresponding steepness of gradient is necessary in any part to give a great depression, for the greatness of the depression arises from the extent of the integration. For instance, in the great polar cyclone of the southern hemisphere, we have seen the greatest eastward velocity, corresponding to the steepest part of the gradient about the parallel of 52°, which is necessary to give the great depression near the south pole, is only twenty-one miles per hour.

22. All barometric oscillations depend almost entirely upon cyclonic action, and are caused generally by the passage of ordinary cyclones over the place of observation. Hence at the equator, where cyclones cannot be formed, there are scarcely any sensible barometric oscillations except the regular small diurnal oscillations. In this case the expression of D in (10) becomes indeterminate, since at the equator ($2\,n \sin l + u$) vanishes, but sec i becomes infinite. But since the value of i depends upon friction and upon the neglected effect of inertia, the value of D must likewise, in this case, depend upon these. Observation shows that at and near the equator the barometric oscillations are extremely small. Col. Sykes observes with regard to the small diurnal oscillations on the plateau of the Deccan: "In many thousand observations made by myself there was not a solitary instance in which the barometer was not *higher* at 9—10 a. m. than at sunrise, *lower* at 4—5 p. m. than at 9—10 a. m., *whatever the indication of the thermometer or hygrometer might be:* nor was there a solitary instance in the year 1830 in which the maximum *night tide* was not higher than the 4—5 o'clock day tide."* Humboldt likewise observes with regard to the regularity of these oscillations in the torrid zone: "This regularity is such that, in day time especially, we may infer the hour from the height of the column of mercury without being in error, on an average, more than 15 or 17 minutes. In the torrid zone of the new continent I have found the regularity of this ebb and flow of the aerial ocean undisturbed either by storm, tempest, rain, or earthquake, both on the coast, and at elevations of nearly 13,000 English feet above the level of the sea." These small diurnal oscillations have nothing to do with the question which we are considering, but as the range of these oscillations is only 0.1 of an inch, the preceding extracts show that the irregular oscillations due to other causes, which in the higher latitudes frequently amount to an inch or more, in the torrid zones are so small as scarcely to interfere with the regularity of the these small diurnal oscillations, and hence must be themselves very small. At the equator there cannot be any gyration, and hence in the storms and tempests of which Humboldt speaks, the motion of the air at the surface of the earth is directly toward the centre of the area of diminished density, and both the inertia of the air and the friction belonging to this motion are overcome by the force arising from almost insensible barometric gradients. The inertia, therefore, of the air merely, which we omitted at the outset in obtaining the relation of (1), is overcome by a very small part of the usual gradients, and its omission, consequently, gives rise to only a very small error in any case. In a cyclone, the force arising from the gradients is almost exactly balanced by that arising from the centrifugal force of gyration, including that of the earth's surface, and so great is the mobility of the air, and so small is the amount of friction, that it is only a very small part of this force which is spent in keeping up the motion of the air between the internal and external parts of the cyclone.

23. If friction is as the first power of the velocity, it is evident from (7), other circumstances remaining the same, that the gyratory motion of a cyclone, and consequently the value of v, is as the sine of the latitude, or sin l, and hence the value of D in (10) is as the square of this sine. The mean monthly ranges of the oscillations of the barometer must, therefore, increase with the latitude, and should be somewhat in proportion to $\sin^2 l$. This is shown to be the case from observation. In the

* "Philosophical Transactions," 1835, page 167.

following table we have the mean monthly ranges given by observation* corresponding to the given latitudes in the first column, and in the last column we have the ranges which increase as $\sin^2 l$, putting the range at the pole equal to 1.6 inches.

1.	Mean monthly ranges.	1.6 in. $\sin^2 l$.
°	*Inches.*	*Inches.*
0	0.1	0.0
30	0.4	0.40
45	1.0	0.80
65	1.33	1.30
78	1.2	1.36

Of course we cannot expect a very nice agreement in the last two columns, since there are several circumstances besides latitude which affect these oscillations. The rapidity of the gyrations in a cyclone, and consequently the amount of barometric depression in the centre, depends very much upon the amount of aqueous vapor with which it is fed, and this diminishes from the equator to the pole. The great cyclones also, which move from lower to higher latitudes, continually increase in diameter, and hence the amount of depression in the centre, and the amount of barometric oscillation at any place caused by the passage of these cyclones over it, must increase toward the poles. These two effects, however, tend in some measure to counteract each other. The monthly range of 0.1 of an inch at the equator is simply that of the diurnal oscillations due to another cause, so that the irregular oscillations sensibly vanish there, as required by theory.

No adequate cause has heretofore been given for these oscillations, nor one which does not equally apply at the equator and in the higher latitudes. We have here given a complete explanation of them, and of their vanishing, or nearly so, at the equator, and of their gradually increasing with the latitude. We now also see the reason why the diurnal oscillations are so regular, not only exactly at the equator, but throughout a belt of considerable extent on each side of it. For the irregular oscillations, which interfere with the regularity of the diurnal, being as the square of the sine of the latitude, must be very small for a considerable distance from the equator. The preceding results also still further verify the law expressed in (1), since the expression of (10), upon which these results are based, is obtained directly from the former.

24. Wherever the atmosphere over any large area of the earth's surface receives a gyratory motion from any cause, this motion gives a value to u and v in (1), and, hence, likewise to G. The term, however, depending upon u, in any such case, is so small in comparison with that depending upon $2n \sin l$, that it can be omitted without sensible error. We can also put $\sec \hat{\imath} = 1$ in this case, and then we have $r = ru$, and r may be either positive or negative. If r is positive, that is, if the gyration is from right to left in the northern hemisphere, or the contrary in the southern, the value of G is positive, and the pressure increases from the centre outward, and there is low barometer at the centre; but if the gyrations are the contrary way in either hemisphere, the value of G is negative, and consequently *high barometer* at the centre of gyration. The westward motion of the air at the parallels of the trade-winds in the Atlantic ocean, and the eastward motion in the middle latitudes, are in some measure obstructed by the continents and deflected from their course. The westward motion north of the equator is turned northward over and along the coast of the United States, and the eastward motion in the north Atlantic is deflected south and north by the continent of Europe, while on the corresponding part of the coast of America the air is drawn away faster than it is supplied by the flow over the continent on account of the greater resistances on land than on sea, and the same occurs on the African coast in the torrid zone. Hence the deflected currents supply these deficiences and give rise to two gyrations in the north Atlantic, the one being a gyration from left to right over a large area having its centre about the parallel of 30° and half-way between the continents; the other, in the northern part of the Atlantic, being a gyration from right to left, and having its centre about the parallel of 65°. Hence there is an area of high barometer in the middle of the former gyration, and of low barometer in the middle of the latter. The effect of these gyrations is indicated by the isobars drawn on the British Admiralty charts, on which several of these isobars, drawn to tenths of an inch of the barometer, always return into themselves. If we suppose these isobars to be 500 miles apart at any place, on the parallel of 30°, the value of G would be 0.02 of an inch, and with this value of G, putting $l = 30°$, we get from (1) $r = 8$ miles, nearly, for the velocity of gyra-

* Loomis's "Meteorology."

tion per hour at that place. In the northern gyration the velocity corresponding to the same gradient would be less on account of the greater value of sin l. From the isobars of the Admiralty chart we see that there are two similar gyrations in the north Pacific, causing an area of high barometer about the latitude of 30°, and of unusually low barometer toward the pole. There are also indications of these gyrations immediately south of the equator in both the Atlantic and Pacific oceans, and also in the Indian ocean, as shown by the isobars, but not toward the poles, since the continents in the southern hemisphere do not extend far enough south to produce the necessary deflections there.

25. If we multiply the second member of (1) by 10,500, the ratio between the density of the air at the earth's surface and mercury, by which we get 800, nearly, for the denominator instead of 8,300,000, G expresses the gradient of sea-level due to the deflecting forces of the earth's rotation. The southern part of the north Atlantic is supposed to make a gyration in about three years, on account of the more complete deflections of the continents in this case similar to those of the atmosphere. This, at the distance of 1,500 miles from the centre of gyration, would give $v = -0.35$ of a mile, v being negative, since the gyrations are from left to right. With this value of v, putting sec $i = 1$, we get from (1) with the new denominator, $G = -0.58$ of a foot for the change of sea-level in 100 miles on the parallel of 30°. The value of G being negative shows that the sea-level is highest in the middle of the gyration, and the difference of sea-level between the centre and the external part of the gyration would amount to several feet. Similar gradients in the sea-level are produced wherever there are ocean currents, the gradient being proportional to the velocity of the current and the sine of the latitude.

No. 5.

CYCLONES, TORNADOES, AND WATER-SPOUTS,

BY WM. FERREL.*

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If all parts of the atmosphere had the same temperature and the same hygrometric state it would remain in a state of static equilibrium. The principal circumstance which disturbs this equilibrium is the difference of temperature between the equatorial and polar regions. This gives rise to an interchanging motion of the air, toward the equator below and from it above, and if it were not for the effect of the earth's rotation on its axis this interchanging motion would be at all places in the direction of the meridian, and would be continually accelerated in its initial motions, until the friction arising from these motions would exactly equal the force producing them, after which the motions of any one place would be constant, but of course, different at different places. The now well-known effect of the earth's rotation is to give rise to a deflecting force to the right of the direction of the moving body in the northern hemisphere and the contrary in the southern, whatever may be the direction of the motion. Hence the air in moving above toward the poles, is deflected toward the east and in moving toward the equator below, toward the west, so that the tendency is for the air to assume an eastward motion in the middle and higher latitudes, and a westward motion nearer the equator. These latter motions, combined with the interchanging motions between the equatorial and polar regions, give rise to what are called the general motions of the atmosphere, depending upon the difference of temperature between these regions and independent of local disturbances of temp erature

The amount of eastward motion depends upon the amount of friction, and must be such that the friction at the earth's surface is equal to the force causing this component of motion, and the same with regard to the westward motions. According to well-established principles of mechanics, there cannot arise any force from the effect of the earth's rotation, which by means of friction would tend to either increase or decrease the earth's rotation, and hence the eastward and westward components of motion must be so adjusted that the sum of all the moments of force acting upon the earth through friction and tending to affect its rotation, must be equal 0, and hence, as there are eastward components of motion in the higher latitude, there must necessarily be westward ones nearer the equator. The eastward motions in the higher latitudes increase with increase of altitude, but nearer the equator the westward motions decrease with increase of altitude and at a certain altitude vanish and become eastward motions.

The deflecting force depending upon the earth's rotation is such that if the air on the parallel of 45° has a velocity of 54 miles per hour, it gives rise to a gradient of pressure, increasing to the right of the direction of motions in the northern hemisphere, and the contrary in the southern, of 0.1 inch of mercury in the distance of one degree of a great circle of the earth. This force, and consequently the gradient, is as the velocity and the sine of the latitude, and hence it is a maximum at the pole and decreases toward and vanishes at the equator. The eastward motion, therefore, in the midde and higher latitudes gives rise to a gradient of pressure increasing *toward* the equator and the westward motion between the tropics and the equator to a gradient of pressure increasing in a direction *from* the equator, and hence there must be a belt of higher pressure all around the globe, having its maximum at the latitude of 30° or 35°, where the dividing line is between the eastward and westward motions. The pressure diminishes from this maximum toward the poles, so that the pressure at the poles, especially the south pole, is less than at the equator. As the southern hemisphere is mostly covered by the ocean, on which the friction is much less than on land, the eastward velocities in the middle and lower latitudes of this hemisphere in their normal state, amount to almost a gale entirely

* "Coast and Geodetic Survey Report for 1878." Appendix 10.

around the globe, and these give rise to a very steep gradient there, and a great barometric depression at the south pole.

The regularity of the general motions of the atmosphere and of the gradients depending upon them, is very much interfered with by irregularities in the distribution of the earth's temperature arising from ocean currents, and from irregularities of the earth's surface, comprising both sea and land with its mountain ranges. This part of the subject was treated in Part I, of these researches, but some knowledge of the principles contained in this part of the subject and of the results is necessary to understand the theory of cyclones, tornadoes, etc.

Cyclones.—Cyclones arise from more local disturbances of temperature. On account of the want of homogeneity of the earth's surface and of the hygrometric state of the atmosphere, the amount of heat received and radiated by the earth's surface and the atmosphere, is very different in different localities. Where more heat is received than radiated, the temperature must continue to rise until the loss of heat by radiation and other means exactly equals the amount received, and hence there cannot be uniformity of temperature even on the same latitudes, and there must be a great many local irregularities in the distribution of temperature independent of the great general disturbance of the equality of temperature between the equatorial and polar regions. These must give rise to corresponding motions of the atmosphere which are superadded to those of the general motions. If in the unequal distribution of temperature it should happen, as it must frequently, that there is a somewhat circular area with higher temperature in the interior and with temperature gradients increasing somewhat regularly on all sides from the centre outward, we should have, at least approximately, the initial condition of a cyclone. There would be a motion of the air from all sides toward the central part of the warmer and more rare air in the interior, a very slow rising up of the air in this part and a flowing out of the air above; that is, there would be an interchanging motion between the colder and warmer parts of the air, just as in the case of the general motions of the atmosphere there is between the equatorial and polar regions, except that in the one case the flow is toward the central part below and from it above, while in the other it is the reverse. Any limited portion of the earth's surface of not very great extent, may be regarded as a plane, and this by virtue of the earth's rotation, has a gyratory motion around its centre, equal to that of the earth's rotation multiplied into the sine of the latitude of this centre. Hence, as in the case of the general motions of the earth, this interchanging motion between the central and exterior part of the warmer and more rarified air, must give rise to gyrations around the centre from right to left in the northern hemisphere, with gyrations the contrary way in the exterior part, and these gyrations in contrary directions must give rise to gradients of pressure increasing in the central part from the centre outward, but in the external part to a gradient of pressure increasing from the outward limit of the gyrations toward the centre, so that there must be a belt of high pressure with its maximum where the interior gyrations in proceeding from the centre, vanish and change signs. These exterior gyrations and the gradients arising from them are generally small in comparison with those of the interior, and they are generally so interfered with by numerous irregularities, that they are not readily shown by observation, but to deny that they exist, would be to deny the truth of a fundamental and well-established principle in mechanics.

The increased pressure under the belt of high barometer surrounding the central part of the cyclone causes a modification of the flow of air toward the centre very near the surface, for the air is forced out from beneath in both directions, the flow toward the outward border very near the surface counteracts and reverses the flow toward the centre arising from the primary and initial cause of disturbance, while the part pressed out on the interior side toward the centre, combines with this flow toward the centre and increases it. For the same reason in the general motions of the atmosphere the flow of air below from the polar to the equatorial regions is reversed very near the surface, and the gentle southwest winds of the middle latitudes are produced.

The preceding condition, found in the unequal distribution of temperature, must be regarded simply as a primary cause of disturbance, giving rise merely to the initial cyclonic disturbances; for without other conditions, depending upon the hygrometric state of the atmosphere, and upon the rate of decrease of temperature with increase of altitude in the atmosphere generally in which the cyclone exists, we would have no cyclone of long continuance or of much violence. With a dry atmosphere, the air in the ascending current of the interior would cool about one degree centigrade for each 100 metres of ascent, so that the air at a very moderate elevation would become colder and more dense than that of the strata of the surrounding atmosphere at the same altitude. The pressure then of

the air at the surface in the interior would become equal to or greater than that of the air generally, unless the rate of decrease of temperature with increase of altitude in the latter were greater than 1° C. for 100 metres, which it never is except in some rare cases and very near the earth's surface only. When this would take place the initial cyclonic disturbances arising from this primary cause of disturbance would cease.

If the air is nearly saturated with aqueous vapor, after ascending to only a moderate elevation, its tension and temperature are so much diminished that the vapor is condensed into cloud and rain, and the heat given out in the condensation of the vapor as the air ascends prevents the rapid cooling which takes place in dry air and the rate of cooling with increase of altitude is reduced, in ordinary temperatures and elevations, to less than half of what it is in dry air. If in this case the rate of decrease of temperature with increase of altitude in the surrounding atmosphere generally is less than that in an ascending current of saturated air, then the temperature of the air in the ascending current, at all altitudes, must be less than that of the air generally, and hence the column of ascending air is lighter than the surrounding air, and the ascending current is kept up as long as it is supplied with air nearly saturated. If, however, after a time, this current comes to be supplied with dryer air, then it has to ascend to a much greater elevation before condensation of the vapor takes place, and as it cools at the rate of 1° C. for 100 metres before it reaches that elevation, it may be cooled down lower than the surrounding air before reaching the elevation where condensation commences, so that if, in this case, we should have the conditions of a continuing cyclone at all, the power of the cyclone would at least be very weak.

Where the state of the atmosphere is such, whether dry or saturated with moisture, that the rate of decrease of temperature with increase of altitude is greater than in an ascending current, it is said to be in a state of unstable equilibrium, since if from any slight predisposing cause such ascending current is once set in motion it must continue until this state is changed, either by the action of what we have called the primary causes of disturbance of temperature or from the inverting action of the currents set in motion. But an atmosphere in this state over a large area would not furnish the conditions for a large cyclone, but there would be simply a bursting up of the lower strata through the upper ones at various places, giving rise to numerous local showers, and often to tornadoes and hail storms. In order to have the complete conditions of a large cyclone it would be necessary to have a central region of warmer and more rarefied air to set in motion ascending currents over a considerable area, and with this there might be considerable cyclonic disturbance if the atmosphere were not quite in the state of unstable equilibrium, but without this latter condition also we could not have a long-continued cyclone. It is seen then that the moisture of the air is a very important element, since without this we cannot have the state of unstable equilibrium unless the rate of decrease of temperature with increase of elevation in the atmosphere generally is greater than 1° C. for each 100 metres, but where the air is saturated this condition takes place with a rate of decrease less than half as great, a rate of decrease which is often found in the atmosphere. The more nearly the air is saturated with vapor, and the greater the decrease of temperature of the air generally with the increase of elevation, the greater is the power of the cyclone. But without these there may be considerable cyclonic disturbance kept up for some time, arising from the primary causes of disturbance, even where the air is so dry that there is very little condensation of vapor into cloud and rain. Professor Loomis has shown that there is sometimes a considerable barometric depression for several days with little or no rain, but in such cases there are only small gradients with no violent winds, and the depression only becomes considerable from the gradients extending over a large area. At the equator, where there is no gyration of the area of rarefaction around its centre in virtue of the earth's rotation around its axis, there cannot be any gyratory motion, but the interchanging motion between the central and external part is entirely radial. Cyclones are therefore never observed on or near the equator.

If there were no friction between the air and the earth's surface, all the conditions of a cyclone could be satisfied by circular gyrations without any radial motions, except in the initial state before the radial motions are brought to rest by means of the friction between the different strata. In this case the linear velocity of the gyrations would be very great near the centre. The greater the amount of friction between the air and the earth's surface the less the velocity of these gyrations, and the greater the inclination of the direction of motion at the earth's surface from the direction of the tangent toward the centre. This is shown by the mathematical expression of this inclination deduced

from the solutions of the equations expressing the conditions of a cyclone, and this same expression shows that near the centre of a cyclone the gyrations of the surface are more nearly circular than at greater distances from it, and that, all other circumstances remaining the same, the nearer the equator the greater the inclination, so that at the equator it becomes 90°, and the motion, as already stated, is radial. In the exterior, or anticyclonal part, where the gyrations are reversed, this inclination at the earth's surface is outward from the tangent. At all altitudes some distance above the earth's surface the friction is small and the gyrations are more nearly circular, but a little inclined toward the centre in the lower part where the interchanging motion is toward the centre, but outward from the centre above, where this motion is from the centre.

If any central area for some reason could be kept colder than the surrounding parts, with a gradient of temperature increasing somewhat regularly from the centre outward, we should have the condition of a cyclone with a cold centre. This condition is furnished in some measure by an island in a northern sea in winter, on which the temperature is less than on the surrounding ocean. In such a case the interchanging motions below and above would be reversed, but the gyrations would be in the same direction around the centre in the interior part as in the case of an ordinary cyclone, and the contrary in the exterior part. The general motions of the atmosphere on each hemisphere of the globe, with the cold poles as their centres, are simply two examples of cyclones of this sort. The gyrations, here in the northern hemisphere, are around the pole from right to left, as in an ordinary cyclone, and the contrary in the southern hemisphere, while at a certain distance from the centre, or pole, these gyrations vanish and change signs, then giving rise to the anticyclonal part of the system, as in an ordinary cyclonic system.

A local cyclone of this sort, with much violence or long continuance, cannot take place. For if there was a central colder area which would give rise to the initial motions of such a cyclone, the air in its descent in the interior would become 1° C. warmer for each 100 metres of descent, and hence the colder initial temperature of the central part would soon be so increased as to equal that of the atmosphere generally surrounding, when the condition giving rise to initial motion would be destroyed and all motion cease. In such a case there would be no advantage in a moist atmosphere, since if it were even saturated as soon as descent in the interior would commence, it would become unsaturated. Hence we never have any violent cyclones of this sort, and nothing more than initial disturbances which continue generally only a short time.

Fixed cyclones.—Where the primary cause of temperature disturbance is fixed to one spot on the earth and kept up continuously, it gives rise to a fixed cyclone. Such an example is furnished by a warm island surrounded by a colder sea. This, unless it were very near the equator, would give rise to a considerable cyclonic disturbance, and, if the island were of considerable extent, to an observable barometric depression. A very remarkable example of such a cyclone exists in the northern part of the Atlantic ocean. Here, on account of the Gulf-stream and the general interchange of waters between the equatorial and polar regions, which tend to equalize the temperatures, there is a considerable area of warmer temperature, especially in the winter season, than that of the surrounding parts, with its centre near Iceland. This gives rise to a fixed cyclone with its interior gyrations around this centre and fixed area of low barometer extending over the greater part of the northern part of the Atlantic ocean. These gyrations on the southern side of this cyclone, combining with those of the general motions of the atmosphere, cause the strong west winds and steep gradients in the middle latitudes of this ocean in the winter. The belt of high pressure of this cyclone is thrown somewhat, on the south side, upon that due to the general motions of the atmosphere at the parallel of about 30° or 35°, and causes the area of high pressure in this ocean at these latitudes.

In the summer season the temperature gradients nearly disappear, and there is very little cyclonic disturbance over this region or barometric depression in the vicinity of Iceland. Very similiar conditions exist in the northern part of the Pacific ocean, but the cyclonic disturbances and the decrease of barometric pressure are not so great.

Progressive motions of cyclones.—Ordinary cyclones, at least soon after their first formation, become independent of local circumstances connected with the earth's surface. The primary temperature disturbance is not sufficiently great and permanent enough to hold the cyclone to the spot where it originates, and it is carried forward by the prevailing general movements of the atmosphere, and the central area of warmer air is maintained by the heat arising from the condensation of the vapor in the interior ascending currents supplied with moist air from the earth's surface by means of

the horizontal currents flowing in from all sides toward the centre. The direction of progressive motion, therefore, is somewhat in the direction of the general motions of the atmosphere in all parts of the earth. Hence cyclones originating near the equator, where there is a westward component of motion are carried westward, but those originating in the middle latitudes, where the general motion of the atmosphere is eastward, are carried toward the east. There is also a tendency of cyclones to move toward the poles where there are no general currents to carry them forward. Cyclones, therefore, which originate in the Atlantic near the equator, are first carried westward and northward toward the West India islands and Florida, until they arrive at the parallel of about 30°, where there is no east or west component of motion, and where, consequently, they move in the direction of the meridian, until they arrive at the middle and higher latitudes, where the general eastward current carries them in that direction, with an inclination still toward the pole. This seems to be the general tendency of cyclones originating everywhere near the equator, but they seem to make their way through toward the pole with greatest facility on the west sides of the Atlantic and Pacific oceans, because there the general motions of the air are deflected around somewhat toward the pole, and aid the cyclones in their progress and carry along a supply of moist air from the equatorial regions for their support. As the power of the cyclone is mostly in the upper cloud region of the atmosphere where the vapor is condensed mostly, the progressive motions of the cyclones depend rather upon the general motions of the atmosphere at considerable altitudes than upon those at the earth's surface. Hence, within the tropics, where the westward motion is small above, the progressive velocity of the cyclone is small, and it is so at the vertex of the parabolic path, where the motion is toward the pole, but after arriving at the higher latitudes where the upper general motion of the atmosphere has considerable velocity, the progressive motion of the cyclone is much accelerated, especially its eastward component.

It must not be supposed, however, that the progressive motion of cyclones depends entirely upon that of the air in which the cyclone exists. It depends also very much upon the direction in which the greatest humidity of the air lies. The progressive motion of the cyclone is generally greater than that of the air, even in the upper regions, and consists rather in the continual formation of new cyclones a little in advance of the old ones, the latter gradually subsiding, and this new formation is mostly likely to occur in the direction of greatest moisture.

Areas of High Barometer.—These arise from the intersecting and overlapping of the circular belts of high barometer of different cyclones both fixed and progressive. In consequence of the gradients arising from the general motions of the atmosphere combined with those of the fixed cyclones and all the other irregularities, the gradients and isobars become very irregular. When to these are added the irregularities of progressive cyclones following and impinging upon one another, this irregularity becomes still much greater, so that it must frequently happen that there are areas in which the barometer stands higher than at any of the surrounding places, just as on a rough sea, where numerous broad waves interfere and cross one another, the surface of the sea has elevations and depressions, not in the form of waves and troughs, but rather of elevated and depressed areas approximating more nearly to a circular form. The isobars of these areas are generally somewhat irregular, but still as they enclose an area, and the winds, according to a well-established law, must blow with a certain not very great inclination to these isobars, the motion of the air is somewhat around these areas in a direction contrary to that of the interior part of an ordinary cyclone. These areas, however, do not form systems of winds complete in themselves, but simply arise from the interference of cyclones, and are therefore not properly called anti-cyclones.

Tornadoes.—These are simply very small cyclones, extending over so small an area that the effect of the earth's rotation has no sensible influence, and the gyrations arise, not from the gyration of this small area around its centre in consequence of the earth's rotation, but from a disturbed state of the atmosphere in which the tornado occurs which renders it impossible for the air to flow from all sides towards a centre without running into gyrations around that centre. This may be illustrated by means of a basin of water with a hole through the bottom in the centre through which the water is allowed to run out. If the water is entirely at rest when the flow commences, there will be only a radial and very gentle motion of the water from all sides toward the centre, without any gyratory motion, but if it has the least gyratory motion in its initial state, even entirely imperceptible, it will run into very rapid gyrations before reaching the centre.

The effect of friction in tornadoes is much less than in cyclones. A cyclone of considerable

extent may be regarded as a disk, with a diameter many times greater than its depth or thickness, and hence the gyrations are very much retarded by friction on the earth's surface; but a tornado is rather a pillar of gyrating air with a very small base in comparison with its altitude, and hence the retardation of the gyrations by friction on the earth's surface in this case is comparatively very small. The gyration of the air, therefore, except near the earth's surface, is very nearly in accordance with the principle of the preservation of areas, and hence the lineal gyratory velocity is very nearly inversely as the distance from the centre, and consequently must become very great near the centre.

In cyclones the barometric gradient and depression of the barometer in the central part are due both to the deflecting force arising from the earth's rotation and the centrifugal force of the gyrations, to the former mostly at a considerable distance from the centre, but to the latter mostly near the centre. In a tornado the diminution of pressure and tension in the centre arises almost entirely from the centrifugal force, that depending upon the earth's rotation being nearly insensible. On account of the rapidity of the gyrations near the centre this diminution of pressure may be very great there, while at a very short distance from the centre it is imperceptible.

Tornadoes occur when, from any cause, the air is in the state of unstable equilibrium already referred to. This may be near the earth's surface, but is most usually up in the region of the clouds, where the air is saturated with moisture and where consequently this state occurs most frequently, since it then requires a rate of diminution of temperature with increase of altitude usually less than half as great as is the case of dry air. When the atmosphere is in this state the air of the lower strata, from any slight disturbance, bursts up through the upper strata at some point, and the higher it ascends the greater is the difference between its temperature and density and those of the surrounding strata at the same elevation, and hence the greater the tendency to rush up at that point. But, as in the case of the basin of water, if the initial state of the air were that of quiescence, there would be only a radial flow of air from all sides toward that point without any gyratory motion or diminution of tension at the centre, and with very little violence of motion. The velocity of the ascending current in this case would not be very great, since the column of ascending air would soon spread out laterally, and become too great. In order to have, therefore, all the conditions of a tornado, it is necessary to have, besides the state of unstable equilibrium, the other conditions which, as in the case of the water in the basin, give rise to gyrations around the central point toward which the air from all sides flows. When these gyrations commence above, as they usually do, since the air there is most frequently in the state of unstable equilibrium, they gradually extend downward, for the gyrations cause a great diminution of tension and of density, and the air consequently in the centre rushes up with great velocity, and that below of the still unagitated strata is drawn in to supply its place, which likewise runs into gyrations around the centre, so that the gyrations in a very short time extend down to the earth's surface. The whole column of gyrating air is like a tall flue containing very rarified air, the centrifugal force of the gyrations acting as a barrier to prevent the inflow of air from all sides into the interior; and if the gyrations at the earth's surface were as rapid as those above, it would be similar to such a flue with all the draught cut off. But very near the earth's surface these gyrations, and consequently the centrifugal force, are very much diminished on account of the friction at the surface, and this allows the air to rush in quite near the surface to supply the draught of the interior ascending current. While, therefore, the gyrations above, on account of the little friction, are almost exactly circular, allowing little air to reach the central part, the motion of the air near the surface is more nearly radial, or at least very much inclined inward from the direction of the tangent. It is the same somewhat in the case of large cyclones. Very near the earth's surface the radial component of motion is much greater than it is at a moderate elevation above, and the inclination from the tangent toward the centre may be very great, while a little above the surface the gyrations are nearly circular. It is readily seen that this must be the case, since the force which overcomes the friction of the gyratory motion depends, in both cyclones and tornadoes, upon the radial component of motion, and hence the greater the friction to be overcome the greater must be this radial component, and where there is little friction, this radial component is very small, and the gyrations nearly circular.

Where the air near the earth's surface is nearly saturated with moisture it has to ascend to only a very moderate altitude at the outer border of the tornado, to have its tension and temperature so reduced that the vapor is condensed into cloud, and nearer the centre, where the tension is diminished by the centrifugal force of the gyrations, the stratum in which condensation and cloud-forma-

tion commences is brought down to the earth at a considerable distance from the centre. In such a case a considerable area of the earth's surface in the central part of the tornado is covered with dense cloud and enveloped in darkness. The indrawing, gyratory and ascensional currents are so strong as to draw in and carry up very heavy bodies and throw them out above to a great distance. Sometimes the ascending current is so strong as to keep a heavy body suspended in the air for a long time until the tornado has progressed many miles, when, after the violence of the tornado begins to abate, the body falls to the earth. Unless the strength of the ascending current is sufficient to carry the body up to an altitude where the air tends outward from the centre, the gradually indrawing currents below that altitude keep the body near the centre and it cannot fall to the earth until the ascending velocity of the current which has carried it up, is diminished.

Water-spouts.—These are simply special cases of tornadoes, as tornadoes are of cyclones. Where the air at the earth's surface in a tornado is not nearly saturated with moisture, it has to ascend to a much greater elevation on the outward border of the tornado before the cloud-formation takes place, and also the nearly horizontal inflowing and gyratory currents below have to approach very near the centre before cloud is formed, and the nearer the earth's surface, the nearer this approach must be. Hence, the base of the cloud assumes a funnel shape above, with a long tapering stem reaching down to the earth or sea. A water-spout, therefore, is simply *the cloud brought down to the earth's surface by the rapid gyratory motions near the centre of a tornado.* This may be explained by means of a deep vessel, instead of a shallow basin, of water, with a hole in the centre of the bottom. If the water is allowed to run out, and it has only an almost perceptible initial gyratory motion, it finally runs into very rapid gyrations around the centre, and the surface of the water and each of the strata of equal pressure under the surface, assume a funnel shape at the top and extend down to the bottom even within the hole, in the form of a long tapering tube. It is the same in the case of the air in a tornado. The fact that the air of the lower strata runs upward through the upper strata, instead of downward through the bottom, does not alter the case, for the gyrations, upon which the lowering of the strata of equal tension and temperature depend, are produced just the same in both cases. The stratum of the air, then, of which the tension and temperature are such as to condense the moisture of the air, assuming this shape, of course the base of the cloud assumes the same. If the dew-point of the air at the earth's surface is 10° C. below the temperature of the air, then air at the outer limit has to ascend about 1,000 metres before cloud-formation takes place, and this determines the height of the spout. The distance from the centre at the base, at which condensation and cloud-formation takes place, depends upon the rapidity of the gyrations, and this upon the amount of initial gyration and of friction. In a tall, slender column of gyrating air the friction is small, and the gyratory velocity may be assumed to be very nearly inversely as the distance from the centre, except very near the centre, where the gyratory velocity becomes almost infinitely great. Without any friction the water-spout would always be brought down to the earth, it might be in the form of a mere thread, however small the initial gyrations, but in nature, where friction, at least near the centre, must diminish considerably the velocity of the gyrations, this is not the case. The diameter of the base of the water-spout depends upon the gyratory velocity, and where this, on account of friction near the centre, is not sufficient to bring the spout down to the surface of the earth, it is seen merely as a funnel-shaped cloud.

Small water-spouts which are seen upon the sea or small lakes in perfectly clear and calm weather, arise from a state of unstable equilibrium of the clear but nearly saturated air near the surface of the water. The principle of their formation is the same, but a greater rate of decrease of temperature with increase of altitude is required, than when their first formation commences up in the region of the clouds.

Cloud-bursts.—We have seen how a heavy body may be sustained and kept up in the air near the centre of a tornado for a long time. In the same manner a large accumulation of rain is sustained, and prevented from being dispersed by the inflowing currents so long as the rain is not carried up where the air flows out from the centre. Calculation shows that the amount of rain condensed from nearly saturated currents of air with such velocities as must exist in the central parts of tornadoes is enormous. The water cannot fall in drops on account of the strength of the current. It therefore accumulates in the body of the cloud, and especially at points where the ascending current is least, until the weight of water becomes so great that it is poured down through the air in streams. Where these streams strike the earth's surface they excavate great holes in the earth, often several yards

deep, and if this occurs on a mountain side, great ravines may be produced. That these holes in the earth and ravines are caused by a stream of water, and not by a very heavy rain, is evident from the fact that the sides of these holes are often cut down almost perpendicularly, while leaves and other light substances, where these holes occur on mountain sides, remain undisturbed near the border on the upper side. The ascending current keeps rain drops from falling, so that no water falls except in the down-pouring streams.

Cloud-bursts are most apt to occur on mountain sides. The tornado, heavily loaded with accumulated rain-water, on approaching a mountain side is very much interfered with by it. The draught of the ascending current, as we have seen, is mostly near the earth's surface. When the base of the gyrating column of air strikes the mountain side, this draught is somewhat cut off, and the whole system somewhat broken up, and the power of the tornado destroyed. Hence the whole accumulation of water is sometimes poured down, almost at once, on the side of the mountain, tearing up rocks and trees, and causing a great ravine.

Hail storms.—As in tornadoes, there is a stratum of air brought down to the earth by the centrifugal force of the gyrations, where the condensation of vapor into cloud and rain first takes place, and which assumes the figure of the water-spout, so very much higher up there is one brought down, it may be entirely to the earth, where the tension is so small and the temperature so low as to freeze the vapor into snow and the rain drops into hail, even in the summer season. The altitude of this stratum, where it is not brought down to a lower level by the gyrations, depends upon the excess of the temperature of the air at the earth's surface above the freezing-point. Drops of rain carried by the ascending current above this stratum, or where it is brought down to or near the earth, within it, are frozen into hail. These may be carried outward above where the ascending currents are so weak that they can fall to the earth, and as they may fall very slowly and may have been cooled down considerably below the freezing-point, they may continue to increase in size all the way down by freezing the water which adheres to their sides in falling, for the ascending current would bring a great deal of rain in small drops and mist in contact with them.

Sometimes much of the hail in thus falling is drawn in toward the centre by the inflowing currents from all sides below, until there is a great accumulation of hail in the centre of the tornado, just as of rain in the case of a cloud-burst. If from any cause, then, the strength of these currents should become suddenly weakened, or the whole system broken up, all this hail would fall rapidly to the earth, and hence the almost incredible amounts of hail which are said to fall sometimes in a very short space of time.

A considerable amount of rain may be carried some distance up into the snow region before it has time to freeze. By the mixture of rain and snow, small balls of very moist snow are formed, which, being carried out where the strength of the ascending current permits them to fall slowly, they continue to grow until they become heavily coated with solid ice, and finally reach the earth. It is in this way that the large hailstones with a snowy kernel within are formed. But these in falling are sometimes carried by the indrawing current below into the central part of the tornado, where the ascending currents are strong enough to carry them up again into the region of soft snow mixed with rain, where they receive another coat of soft snow, less compact than the coat of ice, after which they are thrown out again above where they fall gently down and receive another coat of solid ice. This may be repeated a number of times, the hailstone moving in a sort of oval orbit, upward in the central part, outward above, and down at a distance from the centre where the strength of the ascending current is such as to allow it to fall, and then toward the centre again, to commence another similar revolution. While in the upper snow region it receives a coat of snow, and while in the region of cloud and rain, a coat of solid ice. Hence it is no unusual thing to find large hailstones composed of a number of coatings like an onion, these coatings consisting of alternate layers of frozen soft snow and solid ice.*

Sand-spouts.—These occur mostly on dry, sandy deserts, where the surface becomes very much heated, and the rate of decrease of temperature with increase of altitude is such that the unsaturated and almost entirely dry air is in the state of unstable equilibrium. The sand-spout originates just as any small tornado, or as small water-spouts upon lakes in fair weather, but the air is so dry that there is no condensation of vapor, unless it is at a very great altitude, but the indrawing and ascending currents carry with them a great quantity of dust and other like substances, which assume the

* See "American Journal of Science" II, vol. 50, p. 403.

form of a pillar extending high up into the air. As occurs in all tornadoes and water-spouts, the air flows in from all sides below to supply the draught of the ascending current, mostly near the earth's surface, but also in some degree up to a considerable altitude, and these inflowing currents drive the dust which is raised on all sides in toward the central part, and thus the dusty part of the air assumes the figure of a column.

As the particles of sand gyrate rapidly with the air, the centrifugal force of the gyrations tends to drive the particles from the centre, but this is counteracted by the resistance of the indrawing currents, which is a function of the size of the particle and the strength of their currents, since it is nearly as the square of the product of the velocity of the current into the diameter of the particle. Hence, particles of sand of different sizes arrange themselves at different distances from the centre, the smaller particles penetrating nearer the centre, since the centrifugal force is as the cube of the diameter, while the resistance of the inflowing current is nearly as the square of the diameter. If, however, the particle was very large, it might be kept at so great a distance from the centre, that the ascending current there would not be able to keep it up, so that if there were no limit to the sizes of the particles, yet there would still be a limit to the dimensions of the pillar of sand, which would be determined by the ascending velocity of the air at different distances from the centre.

Water-spouts and sand-spouts are hollow.—Near the centre of the gyrations the centrifugal force is so great that the small particles of condensed vapor in water-spouts, and of fine sand particles in sand-spouts, cannot exist there, or at least they are comparatively rare, so that these spouts have the appearance of being hollow. M. Boué, in the year 1850, observed three water-spouts at the same time on Lake Janina, from top of a high mountain. The weather was entirely clear, without clouds or wind, but very oppressive and hot. The spouts seemed to rise from the lake, and he could look down into the top of them and see that they were hollow in the middle. ("Bulletin Societie Geologique de France," vol. viii, page 274.)

Of a whirlwind observed at Schell City, Missouri, in the summer of 1879, Professor Nipher says: "There were no surface winds strong enough to bear dust along the surface of the ground, but the dust carried up in the vortex was collected only at the vortex of the whirl. The dust column was about two hundred feet high and perhaps about thirty or forty feet in diameter at the top. The direction of rotation was the same as of storms of the northern hemisphere. Leaving the road, the whirl passed out on the prairie, immediately filling the air with hay, which was carried up in somewhat wider spirals, the diameter of the cone thus filled with hay being about one hundred and fifty feet at top. It was then observed also, that the dust column was *hollow*. Standing nearly under it, the bottom of the dust column appeared like an annulus of dust surrounding a circular area of perfectly clear air. The area grew larger as the dust was raised higher, being about fifteen or twenty feet wide when it was last observed." ("Nature," September 11, 1879.)

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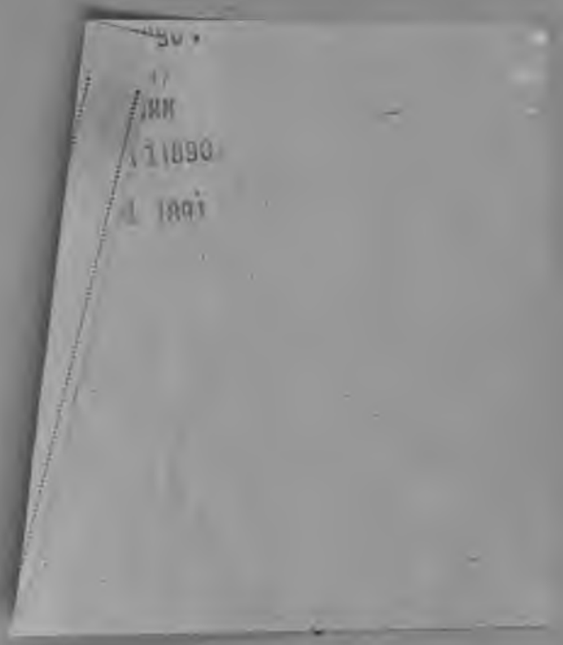
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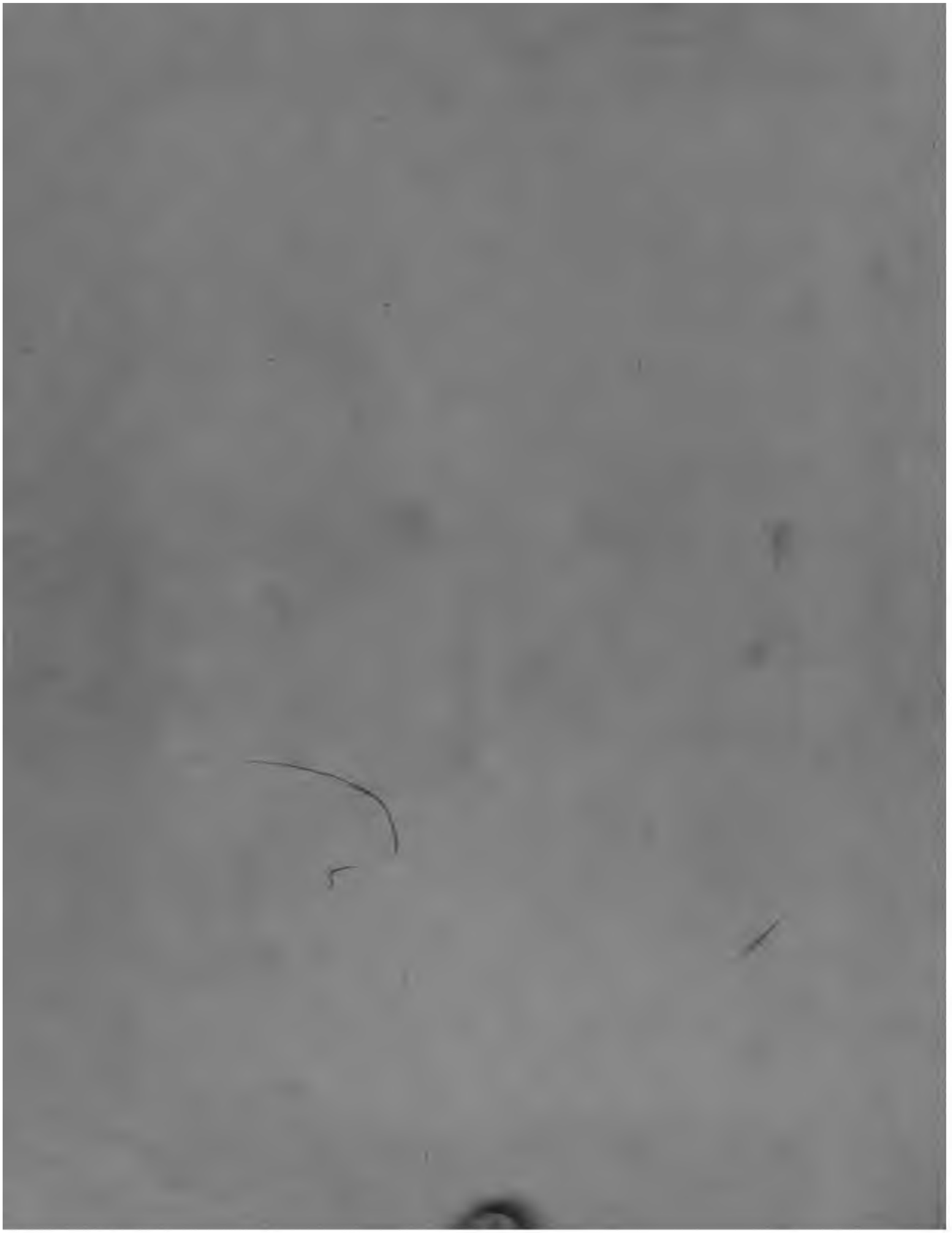

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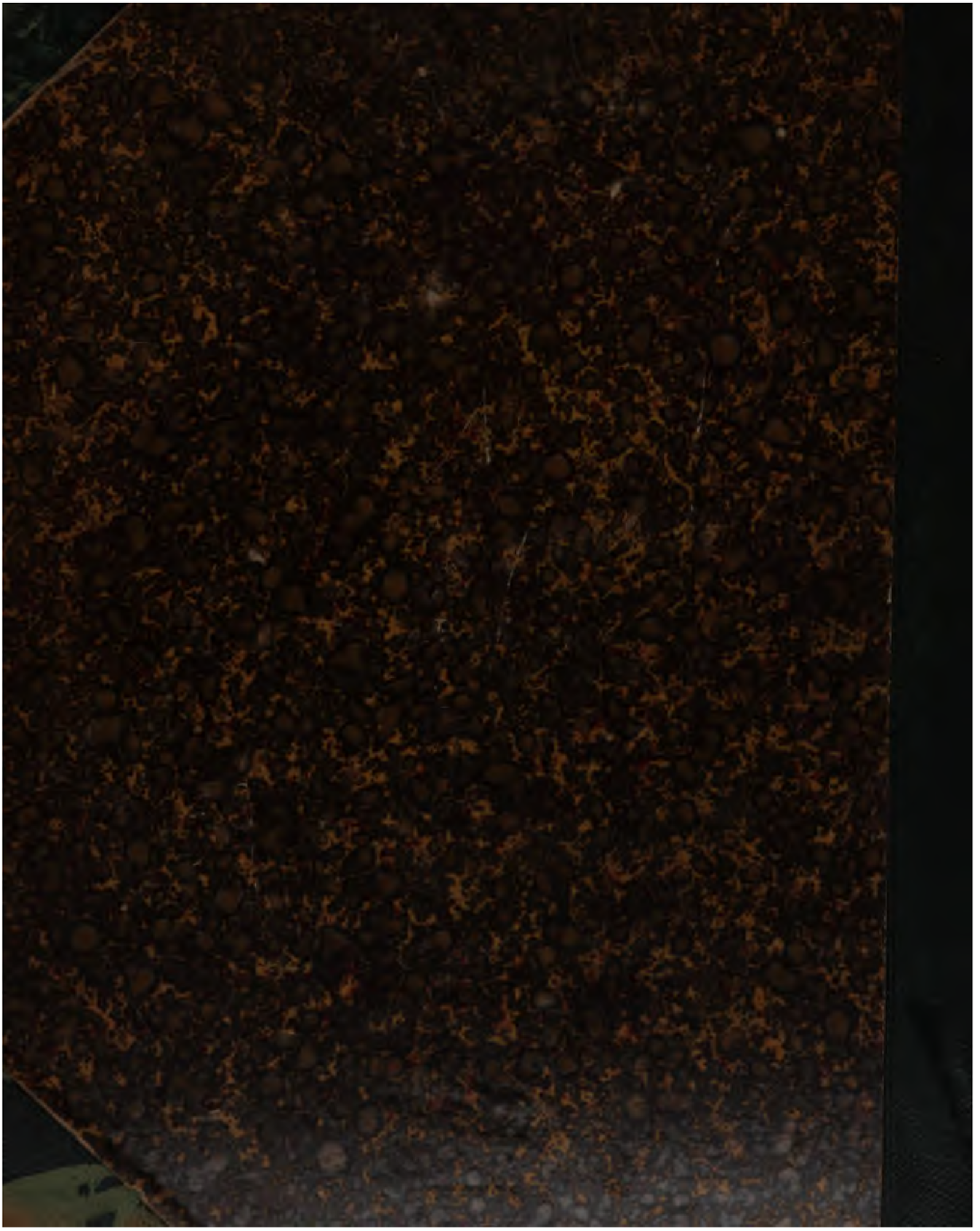
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